Performance of DPSK Diversity Systems

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Abstract - **In this paper we compare several methods of diversity combining for a Rayleigh-faded channel employing DPSK digital signaling. Dependence to the BER of the number of branches is used as the measure of the performances. It is shown that suboptimal diversity schemes, including SC2, SC3 and S + N, and EG combining perform almost identically for dual diversity. It is also shown that S + N model, taking into account the statistical nature of noise, perform slightly gives nearly same performance as SC2 combining but has an advantage because it needs no power measurement.**

Keywords – **DPSK signaling, Rayleigh fading, divesity combining.**

I. Introduction

Binary digital signaling (BPSK, CPSK, DPSK, NCFSK) is often followed by presence of fading. Fading is the term used to describe the rapid fluctuations in the amplitude of the received radio signal over a short period of time caused due to the interference between two or more versions of the transmitted signals which arrive at the receiver at slightly different times. The resultant received signal can vary widely in amplitude and phase, depending on various factors such as the intensity, relative propagation time of the waves, bandwidth of the transmitted signal etc.. A powerful communication receiver technique that provides wireless channel improvement at relatively low cost is a well-known as diversity. Diversity techniques are based on the notion that errors occur in reception when the channel attenuation is large (when chanel is in a deep fade). Supplying to the receiver several replicas of the same information signal transmitted over idependently fading chanels, the probability that all the signal components will fade simultaneously is reduced considerably [2]. There are several techniques for evaluating transmitted signal at the receiver. For the coherent digital signaling (CFSK, BPSK) with independent branch fading, achieved by separating receiver antennas at least 10 wavelenghts, the optimal diversity technique is known as Maximal Ratio Combining (MRC). In Maximal Ratio Combining (MRC), the signals from all the branches are co-phased and individually weighed by feding factor to provide the optimal SNR at the output. But it is seldom implementable in a multipath fading channel because the receiver complexity for MRC is directly proportional to the number of branch signals L available at the receiver. Since L may vary with location as well as time, it is undesirable to have receiver comlexity dependent on a characteristic of the physical channel from a produstion and implementation point of view. Similarly, for the noncoherent digital signaling (NCFSK, DPSK) the commonly used technique is Equal Gain Combining (EGC), where all available branches are equally weighted and then added incoherently. It is clear that this technique is analogous to MRC in the sense that all available branches are used, therefore it has the same undesirable feature of having receiver complexity dependent on L . So it is very desirable to implement some other suboptimal diversity techniques in order to evaluate transmitted signal. The simplest suboptimal technique is the Traditional Selection Diversity Model (SC) that selects, among the L diversity branches, the branch providing the largest signal-to-noise ratio (or largest fading amplitude). Clearly, SC and MRC represent the two extremes in diversity combining strategy with respect to the number of signals used for demodulation. Consequently, other techniques representing compromise between this two were developed. One of them is S + N Selection Model, where S + N denotes a signal-plus-noise sample (i.e., not a power measurement)., and noise is treated as random variable. Also, combining diversity techniques that use two (SC2) or three (SC3) branches with largest amplitudes (or signal-to-noise ratio) for getting transmited signal were developed. In this paper we compare this several diversity combining techniques for a Rayleighfaded channel in the presence of white Gaussian noise (AWGN) employing DPSK digital signaling.

II. System Model

We consider binary differential phase shift keying (DPSK) in slow frequency- nonselective Rayleigh-fading channels with additive white Gaussian noise (AWGN). We assume that there are L independent branches with statistically independent fading processes. If the transmitted signal is $x(t)$, the low-pass equivalent received signal at the i branch [3]

$$\omega_i(t) = \alpha_i e^{j\phi_i} \cdot x(t) + \eta_i(t) \qquad i = 1,2,\ldots,L \tag{1}$$

where

- α_i - fading amplitude (factor) in the i branch (nonnegative number)
- ϕ_i - fading phase in the i branch
- $\eta_i(t)$ - additive complex Gaussian noise in the i branch.

For the coherent digital signaling (CFSK, BPSK) , the optimal diversity technique is known as Maximal Ratio Combining (MRC). Each matched filter output (1) is multiplied by the correspodenting complex-valued (conjugate) channel factor $\alpha_i e^{-j\phi_i}$ and then summed at the combiner. We assumed that noiseless estimates of the complex-valued channel

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parameters (α_i i ϕ_i) were used at the receiver. Since the channel is time-variant, the parameters $\alpha_i e^{-j\phi_i}$ can not be estimated perfectly. Fading fluctuation may be sufficiently fast to preclude the implementation of coherent detection. In such a case, we would consider using either DPSK or FSK with noncoherent detection assuming that fading parameters (α_i i ϕ_i) do not change appreciably over one (NCFSK) or two (DPSK) consecutive signaling intervals. Since the performance (error probability) for NCFSK is the same as that for DPSK with replaced by $\gamma_b / 2$ (3 dB lower required SNR), we will consideronly DPSK digital signaling.

A. EG Combining

For DPSK, EG Combining is the optimal combining technique for feding overcoming . As we said, the channel parameters $\alpha_i e^{j\phi_i}$ remain constant over two consecutive signaling intervals. The output of the EG combiner can be expressed as a decision variable

$$U = Re\left[\sum_{i=0}^{L-1}\left(2E_b\alpha_i e^{j\phi_i} + N_{i2}\right)\left(2E_b\alpha_i e^{-j\phi_i} + N_{i1}^*\right)\right] \qquad (2)$$

where N_{i1} and N_{i2} denote the received noise components at the output of the matched filters in the two consecutive signaling intervals. In such a case, BER is the probability that U is less than zero. The conditional error probability for m combined branches [2]

$$P(\gamma_b) = \frac{1}{2^{2m-1}} e^{-\gamma_b} \sum_{i=0}^{L-1} b_i {\gamma_b}^i \qquad (3)$$

where $\gamma_b = \frac{E_b}{N_0}\sum_{i=0}^{L-1}\alpha_i^2$ is SNR/bit and

$$b_i = \frac{1}{i!}\sum_{n=0}^{L-1-i}\binom{2L-1}{n} \quad . \qquad (4)$$

The probability density function (pdf) of γ_b is

$$p(\gamma_b) = \frac{1}{(L-1)!\,\bar{\gamma}_b^L}\gamma_b^{L-1} e^{-\gamma_b/\bar{\gamma}_b} \qquad (5)$$

where

$$\bar{\gamma}_b = \frac{E_b}{N_0} E\left(\alpha_i^2\right) \qquad (6)$$

is the average SNR/bit. Average BER is computed by averaging the conditional error probability for m combined branches $P(\gamma_b)$ over the probability density function (pdf) $p(\gamma_b)$

$$P_e = \int_0^\infty P(\gamma_b) p(\gamma_b) d\gamma_b \qquad (7)$$

We get a closed-form solution

$$P(e)_{DPSK_{EG}} = \frac{1}{2^{2L-1}(L-1)!(1+\bar{\gamma}_b)^L}\sum_{i=0}^{L-1} b_i (L-1+i)!\left(\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}\right)^i . \qquad (8)$$

B. SC

In the case of the Traditional Selection Diversity Model (SC), the one branch with the largest SNR/bit is selected, the decision statistic is [3]

$$U = Re\left[\left(2E_b\alpha_0 e^{j\phi_0} + N_{02}\right)\left(2E_b\alpha_0 e^{-j\phi_0} + N_{01}^*\right)\right] \qquad (9)$$

where $\alpha_0 = max\{a_i\}$ and N_0 is Gaussian noise with mean equal to zero and variance $2E_b\eta_0$, where η_0 is noise power spectral density. The pdf of γ_b is

$$p_{SC}(\gamma_b) = L\frac{e^{-\gamma_b/\bar{\gamma}_b}}{\bar{\gamma}_b}\left(1 - e^{-\gamma_b/\bar{\gamma}_b}\right)^{L-1} \quad . \qquad (10)$$

Substituting (10) and (3) into (7), with m = 1, we have the average BER for (SC) in Rayleigh channel

$$P(e)_{DPSK_{SC}} = \frac{L}{2}\sum_{k=0}^{L-1}\binom{L-1}{i}(-1)^k \frac{1}{1+i+\bar{\gamma}_b} \quad . \qquad (11)$$

C. S + N

For practical implementations, however, measurement of SNR may be difficult or expensive, especially for high signaling rates. For this reason, the branch with the largest signal-*plus*-noise is often chosen. We use S + N to denote a signal-plus-noise sample (i.e., not a power measurement). When physically realizing this technique , by sampling theoutput of a matched filter, the noise is a random variable (SC assumes that noise is constant in all branches). Consequently, this model perform better than traditional SC model because there is opportunity for at least one sample to be better (less noisy) than the average of the samples. In the case of binary DPSK, the output statistic of the i branch $r_i = Re\left\{\left(2\alpha_i + N_{i,t2}\right)\cdot\left(2\alpha_i + N_{i,t1}^*\right)\right\}$, where Gaussian random variable N_i, $i = 1,2,\ldots,L$ is with zero mean and variance $\sigma^2 = 4/\bar{\gamma}_b$, and $tx, x = 1,2$ represents two consecutive time periods. BER can be expressed as

$$\begin{aligned} \mathrm{P(e)}_{DPSK_{S+N}} &= \sum_{p=1}^{L} Pr\left(max\left\{r_{i,i\neq p}\right\} < r_p , r_p < 0\right) \\ &= L\cdot Pr\left\{\left(max\left\{r_{i,i\neq 1}\right\} + r_1\right) < 0\right\} \end{aligned} \qquad (12)$$

which gives solution

$$\begin{aligned} \mathrm{P(e)}_{DPSK_{S+N}} &= L\cdot\sum_{k=0}^{L-1}\binom{L-1}{k}\frac{(-1)^k}{\left(2\bar{\gamma}_b + 2\right)^{k+1}}\cdot\sum_{i=0}^{k}\binom{k}{i} \\ &\cdot\frac{\left(2\bar{\gamma}_b + 1\right)^{i+1}}{2\bar{\gamma}_b(k-i+1)+k+1} \end{aligned} \qquad (13)$$

D. SC2

Combining diversity techniques that use two (SC2) or three (SC3) branches with largest amplitudes (or signal-to-noise ratio) for getting transmited signal were developed. This techniques, denoted as second or third order selection combining is a compromise between EG Combining and tradititinal SC model and requires a less complex receiver than EG Combining, therefore may be implemented regardless of the number of resolvable branch signals available and, consequently, offer better performance (BER) than traditional SC model. In this case output from modified combiner can be expressed as [1]

$$U = Re\left[\sum_{i=0}^{1}\left(2E_b\alpha_i e^{j\phi_i} + N_{i2}\right)\left(2E_b\alpha_i e^{-j\phi_i} + N_{i1}^*\right)\right] \quad (14)$$

where SNR/bit is $\gamma_b = \frac{E_b}{N_0}\sum_{i=0}^{1}\alpha_i^2$, The pdf of γ_b is

$$p_{SC2}(\gamma_b) = L(L-1)\frac{e^{-\gamma_b/\bar{\gamma}_b}}{\bar{\gamma}_b}\left\{\frac{\gamma_b}{2\bar{\gamma}_b} + \sum_{i=1}^{L-2}\binom{L-2}{i}(-1)^i\frac{1}{i}\left(1-e^{-\gamma_b i/2\bar{\gamma}_b}\right)\right\} \quad . \quad (15)$$

Substituting (15) and (3) into (7), with $m = 1$, we have the average BER for (SC2) in Rayleigh channel

$$\mathrm{P(e)}_{DPSK\ SC2} = \left(\frac{1}{1+\bar{\gamma}_b}\right)^2 \frac{L(L-1)}{8} \cdot\left(\frac{b_0}{2} + \frac{b_1\bar{\gamma}_b}{1+\bar{\gamma}_b} + \sum_{i=1}^{L-2}\binom{L-2}{i}(-1)^i B(i)\right) (16)$$

where

$$B(i) = \frac{(1+\bar{\gamma}_b)\left[b_0(2\bar{\gamma}_b+2+i)+4b_1\bar{\gamma}_b\right]+b_1 i}{(2+2\bar{\gamma}_b+i)^2}. \quad (17)$$

Similarly, in the case of the SC3 model BER is

$$\mathrm{P(e)}_{DPSK\ SC3} = \frac{L(L-1)(L-2)}{64\bar{\gamma}_b}\sum_{m=0}^{2}b_m \cdot\left[\frac{1}{6\cdot\bar{\gamma}_b^2}\cdot\frac{(m+2)!}{(1+1/\bar{\gamma}_b)^{m+3}} + m!\sum_{i=1}^{L-3}\binom{L-3}{i}(-1)^i\frac{1}{i}\cdot V(i,m)\right] \quad (18)$$

where

$$V(i,m) = \frac{(m+1)}{\bar{\gamma}_b(1+1/\bar{\gamma}_b)^{m+2}} - \frac{3}{i(1+1/\bar{\gamma}_b)^{m+1}} + \frac{3}{i\left(1+\frac{1}{\bar{\gamma}_b}+\frac{i}{3\bar{\gamma}_b}\right)^{m+1}} \quad . \ (19).$$

The pdf of γ_b is

$$p_{SC3}(\gamma_b) = \frac{L(L-1)(L-2)}{2}\frac{e^{-\gamma_b/\bar{\gamma}_b}}{\bar{\gamma}_b} \cdot\left(\frac{\gamma_b^2}{6\cdot\bar{\gamma}_b^2} + \sum_{i=1}^{L-3}\binom{L-3}{i}(-1)^i\frac{1}{i^2}\left(i\gamma_b/\bar{\gamma}_b - 3\left(1-e^{-\gamma_b i/3\bar{\gamma}_b}\right)\right)\right) \quad . \quad (20)$$

III. Numerical Results

There are optained numerical results in Fig. 1. for the presented comparison of performances of the several diversity combining tesheniques for a Rayleigh-faded channel employing DPSK digital signaling. Dependence to the BER of the number of branches is used as the measure of the performances for three cases (for different number of branches L =2, 3, i 6). In the case of dual diversity (L = 2) modified selection diversity techniques (SC2, S+ N) and EG combining perform almost identically. This is very notable because the same performance is achieved with the less complex receiver strucure in regard the EG Combining.

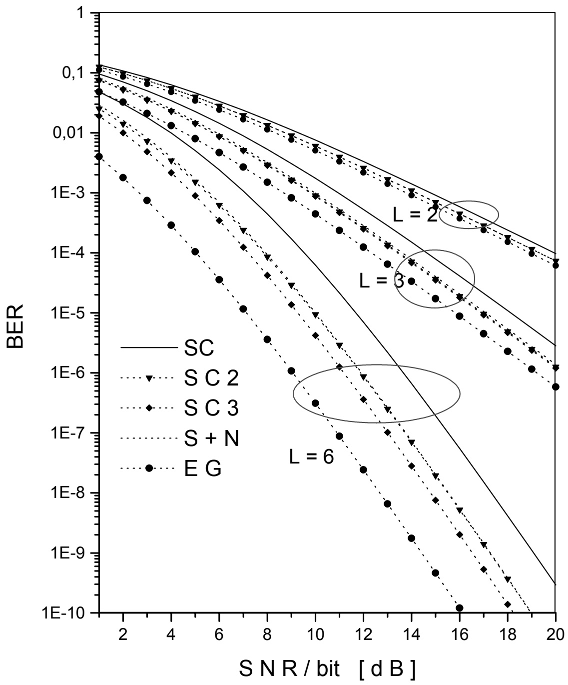

Fig. 1. Performance comparison of DPSK receiver structures for branches L =2, L =3, and L =6 branch diversity

There are also shown that in the case of the L =3 branches SC2, SC3, S + N perform identically, better than the traditional SC model, whilst in the case of L =6 branch diversity SC3 outperforms SC2 and S+N model. In that cases (L > 2) EG model has the best performances. It can be noticed that S + N model is in favor among all modified selection structures because it offers the almost same performances and need no SNR measurement.

IV. CONCLUSION

In this paper we compared several methods of diversity combining for a slow frequency- nonselective Rayleigh-fading channels with additive white Gaussian noise (AWGN). Dependence to the BER of the number of branches was used as the measure of the performances employing DPSK digital signaling. We considered Equal Gain Combining (EG) Combining and Traditional Selection Diversity Model (SC) that represent the two extremes in diversity combining strategy with respect to the number of signals used for demodulation.We also considered modified techniques of selection diversity; S + N Selection Model, where S + N denotes a signal-plus-noise sample (i.e., not a power measurement)., where noise is treated as random variable, and combining diversity techniques that use two (SC2) or three (SC3) branches with largest amplitudes (or signal-to-noise ratio). It was shown in the case of dual diversity(L = 2) modified selection diversity techniques (SC2, S+ N) and EG combining perform almost identically. In the case of EG Combining, receiver complexity is directly proportional to the number of branch signals L available at the receiver, therefore this modified structures are in favor. Finally, in the case when $L > 2$, EG Combining has the best performances. It can be noticed that S + N model is in favor among all modified selection structures because it offers the almost same performances and need no SNR measurement.

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