Application of the Optimal Uniform Polar Quantization on Complex Reflectivity Function

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Abstract - **In compression of polar formatted images, one of the very first steps is to quantize the given polar formatted source data. The goal of this research is to develop a new approach for quantization in sense of efficient data compression and improved image quality. Here we introduce method of the optimal uniform polar quantization as an optimal technique in processing polar formatted SAR images and also as a potential concept in ultrasound diagnostics. An elementary way to relate the viewable SAR image to the energy returns from the observing ground is via complex reflectivity function ξ that relates the incident complex phaser electric field of the transmitter with the reflected field. Another field of implementation is in quantization of measured ultrasound parameter called integrated backscatter (IBS) level that is measure of signal energy. Studies over recent years have analyzed only product polar quantization, but in this paper we will introduce method of optimal polar quantization that provides performance improvement of almost 1 dB in a sense of distortion reduction.**

Key words - **complex reflectivity function, optimal uniform polar quantization.**

I. Introduction

The motivation behind this work is to maintain high accuracy of phase information that is required for some applications such as SAR image processing and digital image processing in medicine, while not losing massive amounts of magnitude information. Both, SAR systems and ultrasound-based methods produce a two-dimensional array of complex numbers that is referred to as an image in rectangular format (i.e., real and imaginary components). These images can also be represented in polar format (i.e., magnitude and phase components). In all previous papers only product polar quantization was considered, but now we can present better results applying method of optimal uniform polar quantization (OUPQ).

Synthetic Aperture Radar (SAR) imagery systems are capable of capturing high-resolution images of large earth areas under all-weather conditions and at any time during day or night. Because of these features, many applications (e.g., terrain mapping and target recognition) benefit from SAR imagery systems.

An inherent significant characteristic of these systems is the generation of large amounts of data and inducing severe constraints related to on board data storage capacities and to communication bandwidth. The fully implemented algorithm in this work covers performance analysis that are driven by aspects related to the operational efficiency such as the lowest distortion achieved and the quality of the restored image based on evaluation of statistical criteria. Under such constraints, the image can only be obtained by capturing and processing the electromagnetic reflections of the earth surface.

SAR images can be represented in polar format that is processed by actual JPEG2000 standard [1], what makes this problem more interesting. Since SAR imaging technologies capture large areas of the earth, statistical properties of these images vary highly in their different regions. To take them into account we propose a variance dependent distortion metric theory (VDDMT).

Classification of stroke causes (atherosclerotic plaque) by means of ultrasound is another potentional field of application for optimal polar quantization [2]. This goal is the development of a new technique for identifying atherosclerotic plaque type that can be implemented in commercial ultrasound scanners, for the purpose of better classifying plaques into categories and to determine stroke risk. The main parameter of interest is a plaque reflectivity "signature", measured by complex reflectivity function.

The concept is based on the fact that the backscatter level from arterial blood (i.e., moving blood) is very nearly constant from person to person. The scattering and attenuating effects of the overlaying, inhomogeneous tissue layer over the plaque region of interest will be investigated for signals from both range cells. An important aspect of this work is the statistical analysis of the integrated backscatter from moving blood.

In this paper we represent method that optimize very quantization of complex reflectivity function. Simple and complete analysis is given for optimal uniform polar quantizer by presenting conditions for the optimality of the polar quantizer and all main equations for optimal phase partitions and optimal number of levels. Performance improvement of OUPQ method over product polar quantization is achievable by allowing different number of phase levels on each magnitude level.

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II. Complex Reflectivity Function and Image Processing

SAR images can be represented in polar format (i.e., magnitude and phase components) so we represent an optimal method for complex reflectivity function quantization in a phase of SAR image compression. If we consider known concept of SAR image formation [1], and if the patch of ground that is observed is small enough so that that its incident intensity is constant over its entirety, then a single parameter defines its scattering properties, which is called the radar cross section σ. The ensemble-averaged intensity at the receiver from a single pulse is the integral of all of the differential intensities where $\overline{\sigma}_0$ is the ensemble mean of an incremental area *dA*. This ensemble mean is given a new symbol σ^0, called the backscatter coefficient and defined as:

$$\sigma^0(\theta,\varphi) = E[\sigma_0(\theta,\varphi)] \qquad (1)$$

The viewable SAR image (real-valued) is derived from the means (σ^0) of the random variables σ_0 in each resolution cell. A more elementary way to relate the viewable SAR image to the energy returns from the ground is via the complex reflectivity function. The complex reflectivity function ξ relates the incident complex phaser electric field of the transmitter with the reflected field:

$$E_{refl}(x,y) = E_{inc}(x,y)\xi(x,y) \qquad (2)$$

where now *(x,y)* refers to particular coordinates on a rectangular geographic grid or pixel locations in the final image. Using this relation and the fact that intensity is directly proportional to the squared magnitude of voltage, the value of σ^0 can also be written as:

$$\sigma^0(x,y) = E\left\{\left|\xi^2(x,y)\right|^2\right\} \qquad (3)$$

The complex valued SAR data represents a linear mapping of the computed ξ. It is the value of, which should be approximated as accurately as possible to allow formation of high quality SAR image.

Almost the same ultrasound-based approach can be introduced for classification of artery arteriosclerosis. Improved atherosclerotic plaque classification will be sought by determining the absolute value of the integrated ultrasound backscatter level, including angle dependence, from the interface between blood and the atherosclerotic lesion, *by using the complex reflectivity function* [2]. As the acoustic wavefront propagates through the inhomogeneous tissue, it is modified in amplitude and phase that has a large and unpredictable effect on the output voltage of the receiving transducer. The ultrasound parameter to be measured is called integrated backscatter (IBS) level that is a measure of reflected signal energy. It is envisioned that a set of absolute backscatter values from different locations inside a plaque and measured under different angles can form a backscatter "signature," to be used in assessing the structure of the plaque.

Due to the randomness associated with the returns received from each incremental area, the underlying complex reflectivity $\xi(x,y)$ is assumed to be a complex random variable, having an unknown probability distribution. Thus the computed complex reflectivity $\overline{\xi}(x,y)$, which is estimated from this received complex voltage signal after correlation processing is also a complex random variable. Since each value in the complex "image" is derived from linear combinations of echo data (via two dimensional convolution with the range and cross range reference functions), the central limit theorem can be invoked to assert the probability density function of the real and imaginary components of each pixel value in the complex image are Gaussian.

Variance-dependent distortion metric theory (VDDMT) is introduced to decrease distortion in polar quantization. In all previous works about polar quantization [1,3-5] only product uniform quantization was always considered (***N***=***P***×***L***) where approximated granular distortion was applied as:

$$D_g^{prod} = \frac{r_{\max}^2}{12L^2} + 2\sigma^2\frac{\pi^2}{3P} \qquad (4)$$

This result shows that while calculating the distortion of a polar quantization scheme the phase distortion should be weighted by $2\sigma^2$ [1] and it makes a base for using VDDMT in improvement of the optimality of polar compression.

We, on the other hand, consider uniform polar quntizer of L magnitude levels and P_i phase reconstruction levels on a magnitude reconstruction level m_i, $1\leq i\leq L$. Polar quantization consists of separate but uniform magnitude and phase N level quantization, so that rectangular coordinates of the source (x,y) are transformed into the polar coordinates in form: $r=(x^2+y^2)^{1/2}$, $\phi=\tan^{-1}(y/x)$ where r represents magnitude and ϕ is phase.

III. Improvement Made by Optimal Polar Quantization

Using polar format of images information about phase can be used as a useful statistical parameter, in physical sense, that corresponds to structural and geometric properties of the scattering medium. Importance of the application of the optimal polar quantizantion is in intention to provide multidimensional information via multiple frequencies. New wavelet-based approaches for efficient compression of complex images use polar format with high reconstruction quality.

In order to find a truly optimal quantizer we have to minimize the distortion, so we proceed as follows:

First we made a partition of the magnitude range $[0,r_L]$ into magnitude rings using L decision levels r_i $1\leq i\leq L$ $(0<r_1<r_2<...<r_L<r_{L+1}=r_{max})$. Magnitude reconstruction levels satisfy obviously $(0<m_1<m_2<...<m_L)$. Then, we make partition of each magnitude ring into P_i phase subpartitions. Let $\phi_{i,j}$ and $\phi_{i,j+1}$ be two phase decision

levels, and let $\psi_{i,j}$ be j-th phase reconstruction level for the i-th magnitude ring, $1 \le j \le P_i$.
Then:

$$\phi_{i,j} = (j-1)2\pi / P_i;\ j = 1,2,...,N_i + 1;,\qquad \text{and}$$

$$\psi_{i,j} = (2j-1)\pi / P_i.$$

Magnitude decision levels and reconstruction levels are given as:

$$r_i = (i-1)\Delta,\ 1 \le i \le L+1$$

$$m_i = (i - 1/2)\Delta,\ 1 \le i \le L$$

At the same time, phase decision levels and reconstruction levels are:

$$\phi_{i,j} = \frac{2\pi(j-1)}{P_i},\quad 1 \le i \le P_i + 1$$

$$\psi_{i,j} = \frac{\pi(2j-1)}{P_i}\quad 1 \le i \le P_i.$$

One UPQ must satisfy the constraint $\sum_{i=1}^{L} P_i = N$ in order to use all of N regions for the quantization. In this paper polar quantizers are designed under additional constraint – that each scalar quantizer is a uniform one. The transformed probability density function for the Gaussian source is

$$f(r,\phi) = \frac{1}{2\pi\sigma^2} \cdot re^{\frac{-r^2}{2\sigma^2}} = \frac{f(r)}{2\pi}. \qquad (6)$$

Without loosing generality we assume that variance is $\sigma^2=1$.
Total distortion D is combination of the granular and overload distortions, $D = D_g + D_o$, where they are

$$D_g = \sum_{i=1}^{L}\sum_{j=1}^{P_i}\int_{\phi_{i,j}}^{\phi_{i,j+1}}\int_{r_i}^{r_{i+1}}[r^2 + m_i^2 - 2rm_i\cos(\phi - \psi_{i,j})] \cdot \frac{f(r)}{2\pi}drd\phi \qquad (7)$$

$$D_o = \sum_{j=1}^{P_L}\int_{\phi_{L,j}}^{\phi_{L,j+1}}\int_{r_{\max}}^{\infty}[r^2 + m_L^2 - 2rm_L\cos(\phi - \psi_{L,j})] \cdot \frac{f(r)}{2\pi}drd\phi \qquad (8)$$

After integration by ϕ and reordering, we have:

$$D_g(P_1,...,P_L) = \sum_{i=1}^{L}\int_{r_i}^{r_{i+1}}[r^2 + m_i^2 - 2rm_i \sin c(\frac{\pi}{P_i})] \cdot f(r)dr \qquad (9)$$

To minimize granular distortion it is necessary to find partial derivations of $D_g(P_i)$. They must be equal to zero. We prove that the problem of minimization of the $D_g(\boldsymbol{P})$ is a convex programming problem. The proof is totally presented in [7] and we are just applying it inhere.

Using method of Lagrange multipliers [7] and applying it on Eq. (9) we are obtain the equation for optimal number of points:

$$P_{iopt} = N\frac{\sqrt[3]{m_i^2 f(m_i)}}{\sum_{j=1}^{L}\sqrt[3]{m_j^2 f(m_j)}};\quad 1 \le i \le L \qquad (10)$$

for fixed N.
Final goal is to find $\boldsymbol{r}_{max}$, , $\boldsymbol{L}_{opt}$, and $(\boldsymbol{P}_{iopt})$, $1 \le i \le L$ for which D_g is minimal. Applying an asymptotical analysis we come to:

$$D_g = \frac{r_{\max}^2}{12L^2} + \frac{9\pi^2L^2\sigma^4}{N^2r_{\max}^2}(1 - e^{\frac{-r_{\max}^2}{6\sigma^2}})^3 \qquad (11)$$

The function $D_g(L)$ is convex of L; from $\frac{\partial D_g}{\partial L} = 0$ we find an optimal solution for L_{opt}:

$$L_{opt} = \frac{r_{\max}/\sigma}{\sqrt[4]{108(1 - e^{\frac{-r_{\max}^2}{6\sigma^2}})^3}}\sqrt{\frac{N}{\pi}} \qquad (12)$$

Finally, the optimal granular distortion for the uniform quantizer is:

$$D_g^{opt} = \frac{\sqrt{3}\pi\sigma^2}{N}(1 - e^{\frac{-r_{\max}^2}{6\sigma^2}})^{3/2} \qquad (13)$$

Using presented method of the optimization we can find expression for optimal product polar quantization; starting from Eq. (5), we come to the result:

$$D_g^{prod\ opt} = \frac{r_{\max}\sigma\pi\sqrt{2}}{6N} \qquad (14)$$

We make an assumption that for the whole picture $\xi(x,y)$ is complex random variable, having Gaussian probability distribution with $\sigma^2=1$. Firstly, picture is divided by method of the OUPQ (using $r_{\max}$ from the literature [6]) into N_1 blocks. The whole picture is divided into L rings and every i-th ring consists of P_i identical blocks.
We define radius of each block $r_{\max}(i)$ corresponding to i-th ring (P_i blocks in ring):

$$r_{\max}(i) = \frac{1}{2}(\Delta r + 2m_i\pi\frac{1}{P_i}),\ \Delta r = \frac{r_{\max}}{L}.$$

Probability function of each ring is:

$P_r(i) = \int_{r_i}^{r_{i+1}}\frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}}dr$, while the block variance of the i-th ring is:

$$\sigma_i^2 = m_i^2 \frac{P_r(i)}{P_i} \qquad (15)$$

It holds that: $\sum_{i=1}^{L} P_i = N_1$.

Number of quantization levels for the i-th block in each ring is:

$$N_i = \frac{N(r_{i+1}^2 - r_i^2)}{r_{\max}^2 P_i} \qquad (16)$$

Applying method of the OUPQ on each block in the ring we can introduce distortion of each single block as:

$$D_i = D_g(N_i, \sigma_i), \quad i=1,L .$$

Actually:

$$D_g^{opt}(i) = \frac{\sqrt{3}\pi\sigma_i^2}{2N_i}(1 - e^{\frac{-r_{\max i}^2}{6\sigma_i^2}})^{3/2} \qquad (17).$$

Optimal granular distortion under product quantization is:

$$D_g^{prod}(i) = \frac{r_{\max}(i)\sigma_i\pi\sqrt{2}}{6N_i} \qquad (18)$$

Total distortion can be defined as:

$$D_{tot} = \sum_{i=1}^{L} D_i P_r(i) . \qquad (19)$$

Gain $G(i)$ provided by OUPQ method for each block i over optimal product quantization is:

$$G(i) = 10\log(\frac{D_g^{prod}(i)}{D_g^{opt}(i)}) = 10\log(\frac{r_{\max}(i)\sqrt{2}}{3\sqrt{3\sigma_i}(1 - e^{\frac{-r_{\max}^2(i)}{6\sigma_i^2}})^{3/2}})$$

In order to perform the improvement obtained by our method we'll present an example: in Table I are given results for gains $G(i)$ and average total gain G_{tot} for different N (and corresponding optimal number of levels). Average total gain can be defined as:

$$G_{tot} = 10\log\frac{D_{g\,tot}^{prod}}{D_{g\,tot}^{opt}} \qquad (20)$$

Result show that total gain is approximately about 0.9dB for OUPQ over product quantization.

Another improvement that is achievable by optimal polar quantization is in a fact that vector quantizers can provide magnitude and phase quantization together and adaptively, while VDDMT can be applied on other data types, such as DFT coefficients.

IV. Conclusion

The design of optimal uniform polar quantization (OUPQ) method for distortion minimization problem is presented in image processing applying it on complex reflectivity function as possible to measure parameter, both in SAR systems and the ultrasound diagnostic. In order to improve quantization efficiency we introduced variance-dependent distortion metric theory (VDDMT). Significant performance improvement is shown through average total gain of about 0.9dB over common product quantization.

TABLE I

Gain $G(i)$ for different N and average total gain G_{tot}

i	N_1=128, L_{opt}=7	N_1=260, L_{opt}=10	N_1=512, L_{opt}=15
1	5.65367	10.9809	8.18636
2	1.2459	5.16583	3.44628
3	0.693069	3.29101	1.49162
4	0.794187	2.38799	0.785675
5	0.717837	2.06908	0.692334
6	0.787852	2.16667	0.757765
7	1.83238	2.63826	0.795897
8		3.47915	0.764876
9		4.67662	0.702233
10		6.25186	0.708491
11			0.945527
12			1.53701
13			2.50642
14			3.75988
15			5.17384
G_{tot}	0.9170410dB	0.930116dB	0.933998dB

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