Wireless System Capacity Increase by Using MIMO Channels with st Codes

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Abstract: **Subject of this paper is investigation of MIMO systems performances improvement. Channel parameters are considered to be known both to the transmitter and receiver. Practical realizations with use of some adaptive methods are presented. Capacity and BER obtained with these realizations are compared with optimal results.**

Keywords: **MIMO channels, space-time codes, beamforming.**

I INTRODUCTION

Available radio spectrum is more and more occupied because of enormous use of wireless digital telecommunication systems and transfer of multimedia content with demands for high data rates. For that reasons, there is a great need for method that would enable significantly greater capacities by increasing spectral efficiency. One of best ways for increasing channel capacity is use of MIMO (*Multiple Input Multiple Output*) systems. In MIMO systems, there are multiple antennas on transmit and receive part, so signals are combined in optimal way, obtaining better channel capacity and BER performances. By using of space-time codes, space diversity and error control coding give unified effects.
This way of data transfer was originally designed for growing wireless LAN systems. Some space code realizations are in 3GPP proposal for UMTS systems [3].
General model of MIMO systems with space-time codes is described in this paper. Overview of obtained improvements and comparative analysis of standard and adaptive techniques for channel capacity improvement is also given.

II SYSTEM MODEL

MIMO system model with n_T transmit antennas and n_R receive antennas is considered. This system model can be described with following equation

$$y = H\cdot x + n, \qquad (1)$$

where $n_R\times 1$ column vector y is output signal, $n_T\times 1$ column vector x is input signal, H is $n_R\times n_T$ channel matrix, and elements of $n_R\times 1$ column vector n are uncorrelated Gaussian variables. In that case $R_{nn} = \sigma^2 I_{n_R}$.
In this paper it is considered case when there is no line of sight between transmitter and receiver, so channel matrix has Rayleigh coefficients. Total signal power is P_T and it doesn't depend on number of transmit and receive antennas. Channels that are narrow enough are considered in analysis, so fading can be considered frequency flat.

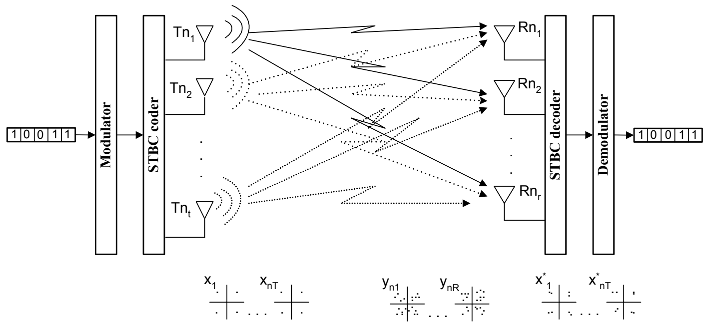

Fig. 1. MIMO channel block scheme

One date flow is divided into several flows, which are taken to a larger number of antennas (Fig 1). With such a realization both space and time multiplexing are achieved.
In this analysis realization of H is known to the receiver. Receiver can simply estimate channel transfer matrix, by using pilot sequences.
In cases when transmitter should have channel information, the system should have highly reliable reverse channel.
Depending on does the reverse channel exists or not, MIMO systems can be classified as *open-loop* (not adaptive) or *close-loop* (adaptive) systems.

III OPEN LOOP MIMO SYSTEMS

In open loop MIMO systems channel parameters are known to the receiver but not to the transmitter. It is supposed that signal vector x on transmit part has n_T statistically independent signals, what means that covariance matrix on transmit antennas is:

$$R_{xx} = \frac{P_T}{n_T} I_{n_T} \qquad (2)$$

This case is known as *optimal blind signaling* or *full open loop*, and channel capacity is given by following equation:

$$C = W \log_2 \left[\det\left(I_{n_R} + \frac{P_T}{n_T \sigma^2} H H^T \right) \right] \qquad (3)$$

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where σ^2 is noise variance on each of n_R receive antennas.
Previous equation determines channel *instantaneous capacity*, for given realizations of channel matrix *H*. It is possible to define *average capacity*, as average value of instantaneous capacities for all possible realizations of matrix *H* and *outage capacity*, as capacity that cannot be achieved with probability P_0.
Complementary cumulative distribution capacity (*ccdf*) for various values of SNR and various numbers of transmit and receive antennas is presented in Fig. 2.
Capacity of radio system is approximately linearly proportional to dimension of the system (number of transmit and receive antennas) [3].

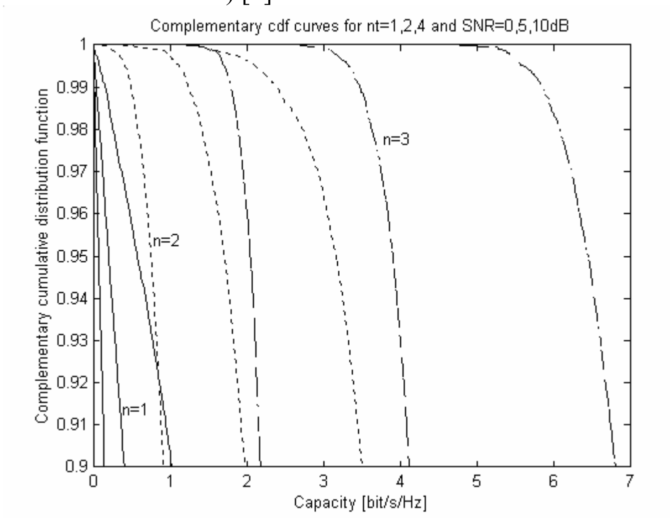

Fig. 2. Complementary cumulative distribution function for SNR=0dB, 5dB, 10dB and n_T=n_R=1, 2, 4.

As condition in Eq. (2) almost never completely holds, Eq. (3) is just theoretical limit of MIMO system spectral efficiency.
Space-time codes are probably the most perspective technique that is applied with aim of obtaining bit rates that are close to this theoretical limit. Space-time block codes, STBC are especially popular, because they have constant code word length and therefore are easy to be decoded.
There are various realizations of STBC and the most popular among them is the one that Alamouti proposed [4]. In this realization, by use of two antennas on transmit and receive part, it is formed orthogonal block code with code ratio equals one. That is basic advantage of this scheme. For complex signal constellations other codes exists too, but all of them have code ratio that is smaller than one.
Transmission matrix for Alamouti code is

$$G = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix} \qquad (4)$$

It can be shown [1] that the capacity obtained with implementation of orthogonal STBC with code ratio *k/L* is given with:

$$C = W\frac{k}{L}\log_2\left(1+\frac{P_T}{n_T\sigma^2}\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\left|H(i,j)\right|^2\right) \qquad (5)$$

IV Closed Loop MIMO Systems

In closed loop MIMO systems, channel state is known at both transmit and receive part, so these information are used for adaptive capacity improvement methods. In this part of the paper adaptive techniques known as *water-filling* i *beamforming* will be shown.
By the singular value decomposition theorem any channel matrix *H* can be written in form $H=UDV^*$, where *U* and *V* are unitary matrices, and *D* is nonnegative diagonal matrix. The diagonal entries of *D* are nonnegative square roots of the eigen- values of matrix HH^H. With introduction of following transformations:

$$y' = U^H y,\;\; x' = V^H x,\;\; n' = U^H n, \qquad (6)$$

Equivalent MIMO channel is

$$y' = Dx' + n' \qquad (7)$$

The number of non-zero eigenvalues of matrix HH^H is equal to rank *r* of matrix *H*, and its maximal possible value can be $\min(n_T, n_R)$.
Previous equation can be written in scalar form:

$$y_i' = \sqrt{\lambda_i}x_i' + n_i', \text{ i= 1, 2, …, r}$$

$$y_i' = n_i', \text{ i= r+1, r+2,…, } n_R \qquad (8)$$

From previous equations it can be concluded that equivalent MIMO channel consists of *r* independent parallel channels that are called *channel eigenmodes*.
In *water-filling* techniques total transmit power is distributed to a number of antennas depending on channel transfer matrix eigenvalues.
Subchannels whose appropriate eigenvalues are greater ensure more quality data transfer, so they should have greater part of total transmit power.

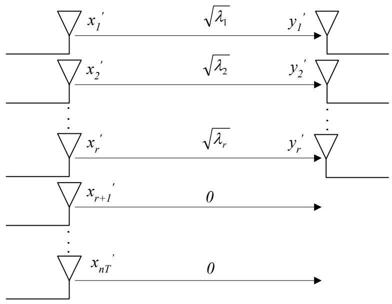

Fig. 3. Equivalent MIMO channel

Power of i-th mode is

$$P_i = \left(\mu - \frac{\sigma^2}{\lambda_i}\right)^+ \text{ i=1, 2, …, n} \qquad (9)$$

where $x^+=\max(0, x)$, and μ is determined so equation $\sum_{i=1}^{n_T} P_i = P_T$ is satisfied
So, capacity that can be obtained with use of *water-filling* procedure is given with

$$C = W\sum_{i=1}^{n_T}\log_2\left(1+\frac{P_i\lambda_i}{\sigma^2}\right) \qquad (10)$$

Water-filling procedure adjust power of each mode depending on λ, so C in great measure depends on eigenvalues distribution. Eigenvalues distribution can be obtained by calculating an average value for a great number of matrix H realizations. Eigenvalues distribution for system with n_T=n_R=4 determined from 5000 realizations of matrix H is presented in Fig. 4.

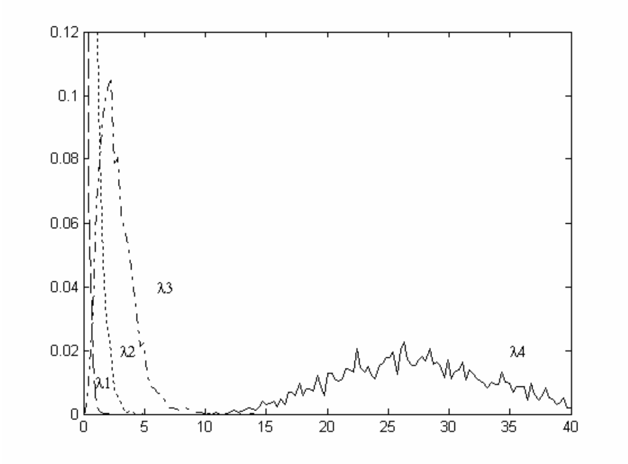

Fig. 4. Eigenvalues distribution of HH^H matrix

In the case of Rayleigh channel there is almost always one component that is dominant, meaning that there is one mode that provides significantly better transfer.

Beamforming techniques are based on transfer through mode with maximal value of λ, while other modes are not used at all. In these techniques adaptation algorithm is very simple, because it is enough to scale modulated signal on coder input with weight coefficients and to sent them to transmit antennas $y=H\cdot w\cdot x+n$.

Transfer through maximal mode is obtained if weight coefficient vector w is eigenvector of matrix HH^H that correspondents to its maximal eigenvalue.

Maximal possible obtained capacity with *beamforming* techniques is given with:

$$C = W\log_2\left(1+\frac{P_T\lambda_{\max}}{\sigma^2}\right) \qquad (11)$$

V Numerical Results

Subject of this paper is comparation of characteristics of optimal *open loop* and *close-loop* methods (*optimal blind signaling* and *water filling*) with concrete realizations – *Alamouti* scheme as suboptimal *open-loop* system and *beamforming* scheme as suboptimal *close-loop* system.

Comparation of maximal possible obtained capacities is done by using Eqs (3), (5), (10), (11). Channel capacity ccdf is formed after simulation of 20000 matrix H realizations. After that, channel capacities with 1% outage probability are determined. Results are given in Figs 5. and 6:

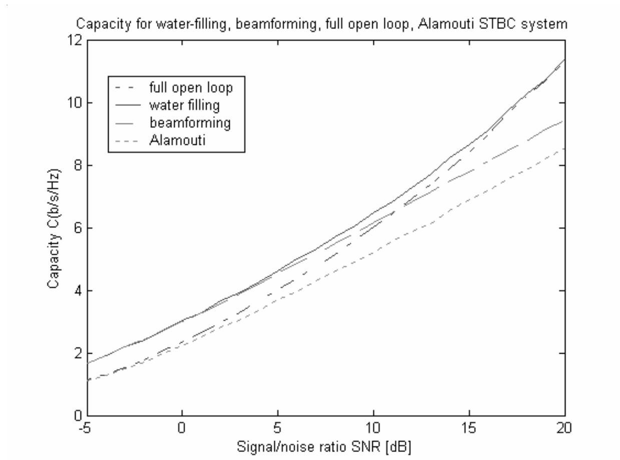

Fig. 5. Capacity comparation for system with n_T=n_R=2

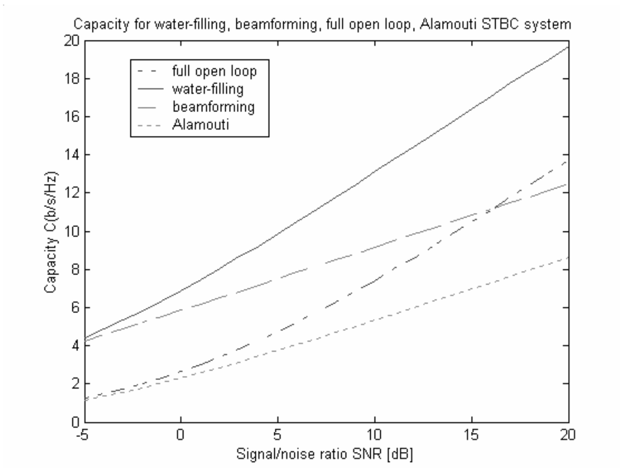

Fig. 6. Capacity comparation for system with n_T=16,n_R=2

Suboptimal techniques, *beamforming* for *close-loop*, and *Alamouti* STBC for *open-loop*, are equivalent with appropriate optimal solution for small values of SNR. Difference between suboptimal and optimal solutions rises for greater values of SNR.

In case of symmetric systems, capacities for *water-filling* and *full open loop* are equal for greater SNR values, and no improvement can be obtained with adaptation. It is not case with asymmetric systems with great number of transmit antennas, where performances of *water-filing* systems are more superior.

Beamforming system, for both symmetric and asymmetric systems, in comparation with *Alamouti* scheme with same number of transmit and receive antennas obtains constant gain. Optimal solutions are very difficult for realizations so these systems are most frequently used in practice.

BER is estimated with *Monte Carlo* simulation. In this simulation 50000 information bit array is generated, signals are modulated and space-time coded with appropriate techniques. Rayleigh fading channel (σ=1) with effect of AWGN (SNR from –2 to 10dB) is considered. Following systems are compared: 2x2 system with *beamforming*, 2x2 *Alamouti* STBC and 1x1 system without correction of Rayleigh fading.

Simulation results are shown in Figs. 7 and 8, respective.

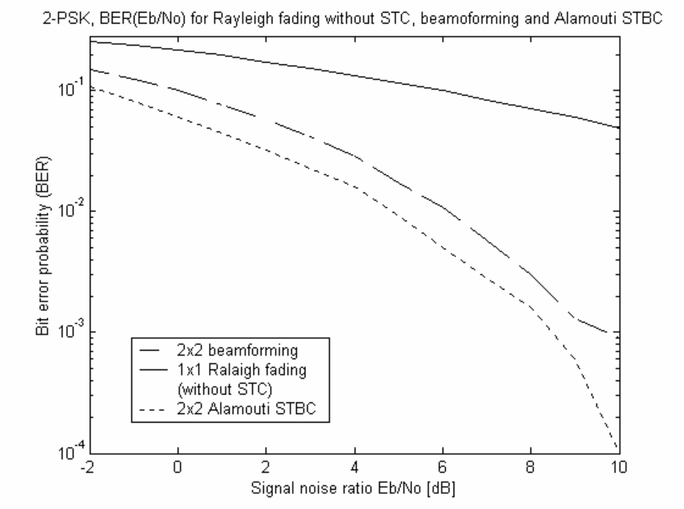

Fig. 7. BER comparation for BPSK

It can be concluded from figures that implementation of *Alamouti* STBC code can significantly improve performances of systems with effects of Rayleigh fading. This improvement is better for greater values of SNR. These performances are getting worse for denser signal constellations.

Beamforming, which represents adaptive techniques, ensures constant system performances improvement. In case of BPSK for all considered SNR values gain is in extent 2-3 dB. Gain is smaller for denser signal constellations, as in case of not adaptive *Alamouti* system. For great SNR values this techniques is not better than *Alamouti* scheme, and it agrees with conclusion that *beamforming* gives best results for small SNR values.

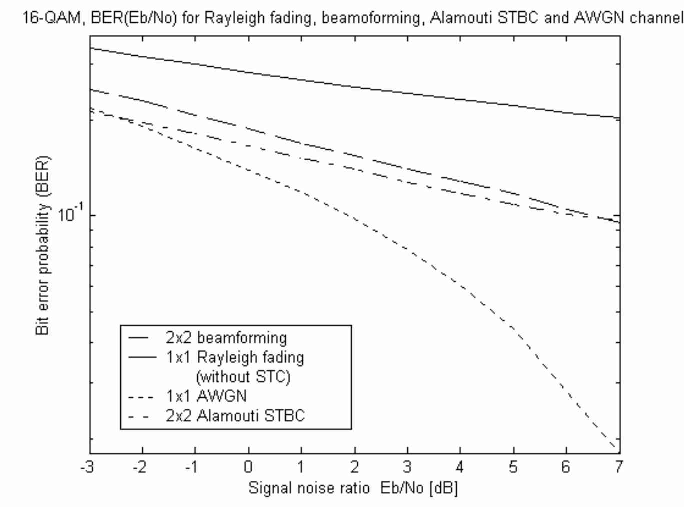

Fig. 8. BER comparation for 16-QAM

VI Conclusion

In this paper it is shown that MIMO systems capacities are much better than standard systems capacities. In theory, capacity is approximately linear proportional to dimension of MIMO channel. The aim is obtaining this theoretical limit in practice.

It is shown that with use of adaptive techniques better performances are obtained, especially in case of asymmetrical systems.

Simulation results shows that for small SNR values characteristics of suboptimal methods are relatively close to optimal, while this difference is significant for greater SNR values.

It can be concluded that in wireless digital communications area, with use of appropriate adaptive systems and STC, it is possible to obtain significantly greater capacities, close to those predicted by theory of MIMO systems. The use of these systems offers higher data rate and more reliably data transfer, what makes them suitable for multimedia data transfer.

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