Noncoherent Detection FSK with Envelope Detector where Fading Present

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Abstract – In this paper, we analyse noncoherent detection of FSK digital signal where feding is present. Fading arises because signal is transmited over more ways. For that, equivalent amplitude of signal on input of receiver is random. It is determinated error probability of digital telecommunication system where is signal FM and amplitude of signal takes Rayleigh distribution.

Keywords – Rayleigh distribution, the fading, noncoherent detection FSK.

I. INTRODUCTION

In many transmission media, the received signal is composed of vector sum of many sine waves whose phases change slowly with time. Example of “multipath” channels which experience such behavior include propagacion via the troposphere, ionosphere, and radar backscatter from complex targets. The vector sum signal will have an envelope which changes with time and is called a fading signal. Since the signal is the sum of many sine waves it is reasonable to model it as a Gaussian random process. If this is the case, then the envelope is Rayleigh distributed-hence Rayleigh fading.

II. FUNCTION OF DENSITY DISTRIBUTION AND SYSTEM ANALYSIS

Figure represents model of FSK system for noncoherent detection with envelope detector:

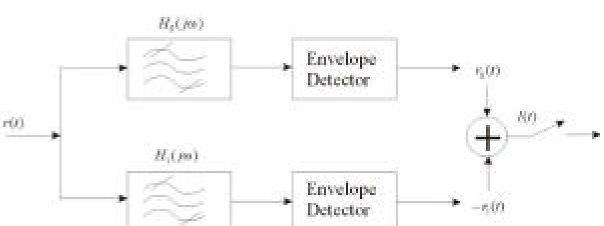

Figure 1. Model of noncoherent FSK receiver

Signal on input of receiver:

$$H_0 : r(t) = A\cos(w_0 t + j) + n(t) \quad (1)$$

$$H_1 : r(t) = A\cos(w_1 t + j) + n(t) \quad (2)$$

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Phase j is uniformly distributed over the interval $(0,2p)$, until amplitude has Rayleigh distribution:

$$p(A) = \frac{A}{s^2} \cdot e^{-\frac{A^2}{2s^2}} \quad (3)$$

Signal $r_0(t)$, for hipothesis H_0 i H_1, is:

$$r_{00}(t) = \\ = \sqrt{[A\cos j + x_0(t)]^2 + [A\sin j + y_0(t)]^2} \quad (4)$$

$$r_{01}(t) = \sqrt{x_0^2(t) + y_0^2(t)} \quad (5)$$

Signal $r_1(t)$, for hipothesis H_0 i H_1, is:

$$r_{10}(t) = \sqrt{x_1^2(t) + y_1^2(t)} \quad (6)$$

$$r_{11}(t) = \\ = \sqrt{[A\cos j + x_1(t)]^2 + [A\sin j + y_1(t)]^2} \quad (7)$$

Signal $l(t)$:

$$H_0 : l(t) = r_{00}(t) - r_{10}(t) \quad (8)$$

$$H_1 : l(t) = r_{01}(t) - r_{11}(t) \quad (9)$$

To take into consideration that variance of process $x_1(t)$, also $x_0(t)$, equal s, and to take into consideration that liner operation of averaging, variance process $x_1(t) - x_0(t)$ is:

$$\overline{[x_1(t) - x_0(t)]^2} = 2s^2 \quad (10)$$

Now, we have density distribution is:

$$p_1(r_{00}) = \frac{1}{s^2} \cdot e^{-\frac{r_{00}^2 + A^2}{2s^2}} \cdot I_0\left[\frac{r_{00}A}{s^2}\right] \quad (11)$$

$$p_2(r_{01}) = \frac{1}{s^2} \cdot e^{-\frac{r_{01}^2}{2s^2}} \quad (12)$$

$$p_3(r_{10}) = \frac{1}{s^2} \cdot e^{-\frac{r_{10}^2}{2s^2}} \quad (13)$$

$$p_4(r_{11}) = \frac{1}{s^2} \cdot e^{-\frac{r_{11}^2 + A^2}{2s^2}} \cdot I_0\left[\frac{r_{11}A}{s^2}\right] \qquad (14)$$

Conditional error probability function is:

$$p_0(l/A) = \int_{-\infty}^{\infty} p_1(x+l)p_3(x)dx$$
$$= \int_{-\infty}^{\infty} \left(\frac{x+l}{s^2} \cdot e^{-\frac{(x+l)^2 + A^2}{2s^2}} \cdot I_0\left[\frac{(x+l)A}{s^2}\right] \cdot \frac{x}{s^2} \cdot e^{-\frac{x^2}{2s^2}} dx \right) \qquad (15)$$

$$p_1(l/A) = \int_{-\infty}^{\infty} p_1(x+l)p_3(x)dx$$
$$= \int_{-\infty}^{\infty} \left(\frac{x+l}{s^2} \cdot e^{-\frac{(x+l)^2}{2s^2}} \cdot I_0\left[\frac{x \cdot A}{s^2}\right] \cdot \frac{x}{s^2} \cdot e^{-\frac{x^2 + A^2}{2s^2}} dx \right) \qquad (16)$$

Average of $p_0(l/A)$ and $p_1(l/A)$ is obtained by averaging over all A:

$$p_0(l) = \int_0^{\infty} p_0(l/A)p(A)dA$$
$$= \int_{-\infty}^{\infty}\int_0^{\infty} \left(\frac{x+l}{s^2} \cdot e^{-\frac{(x+l)^2 + A^2}{2s^2}} \cdot I_0\left[\frac{(x+l)A}{s^2}\right] \cdot \frac{x}{s^2} \cdot e^{-\frac{x^2}{2s^2}} \cdot \frac{A}{s^2} \cdot e^{-\frac{A^2}{2s^2}} \right) dxdA \qquad (17)$$

$$p_1(l) = \int_0^{\infty} p_1(l/A)p(A)dA$$
$$= \int_{-\infty}^{\infty}\int_0^{\infty} \left(\frac{x+l}{s^2} \cdot e^{-\frac{(x+l)^2}{2s^2}} \cdot I_0\left[\frac{x \cdot A}{s^2}\right] \cdot \frac{x}{s^2} \cdot e^{-\frac{x^2 + A^2}{2s^2}} \cdot \frac{A}{s^2} \cdot e^{-\frac{A^2}{2s^2}} \right) dxdA \qquad (18)$$

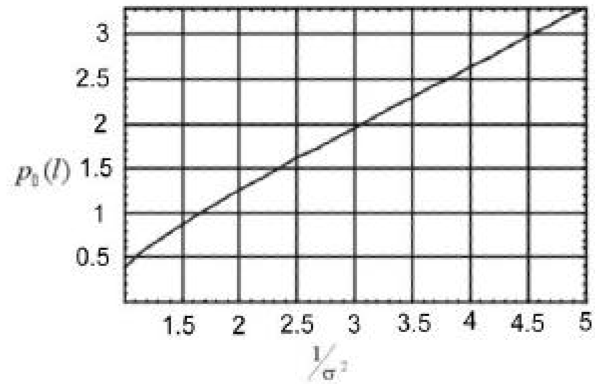

Figure.2. Distribution envelope $p_0(l)$, for Rayleigh fading signal

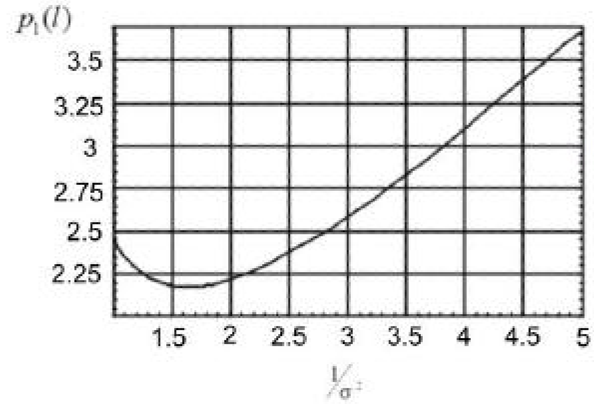

Figure.3. Distribution envelope $p_1(l)$ for Rayleigh fading signal

Finally, total error probability is:

$$P_e = P(H_0) \cdot P(H_1/H_0) + P(H_1) \cdot P(H_0/H_1) =$$
$$= P(H_0) \cdot \int_{-\infty}^{\infty}\int_0^{\infty} \left(\frac{x+l}{s^2} \cdot e^{-\frac{(x+l)^2 + A^2}{2s^2}} \cdot I_0\left[\frac{(x+l)A}{s^2}\right] \cdot \frac{x}{s^2} \cdot e^{-\frac{x^2}{2s^2}} \cdot \frac{A}{s^2} \cdot e^{-\frac{A^2}{2s^2}} \right) dxdA$$
$$+ P(H_1) \cdot \int_{-\infty}^{\infty}\int_0^{\infty} \left(\frac{x+l}{s^2} \cdot e^{-\frac{(x+l)^2}{2s^2}} \cdot I_0\left[\frac{x \cdot A}{s^2}\right] \cdot \frac{x}{s^2} \cdot e^{-\frac{x^2 + A^2}{2s^2}} \cdot \frac{A}{s^2} \cdot e^{-\frac{A^2}{2s^2}} \right) dxdA \qquad (19)$$

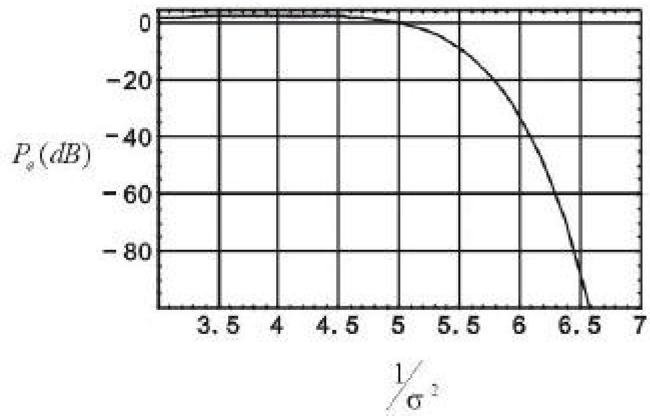

Figure.4. Error probability for signal/noice $1/s^2$

III. CONCLUTION

In this paper, it is analysed digital telecommunication system where is signal FSK modulation. In same cases, signal level of useful signal is not stable on input receiver. In that way, it arises appearance which is called fading. In practice, the fading arises between transmitters and receivers, which not have optical visibility, on short waves where is reflected via the troposphere, ionosphere, and radar backscatter from complex targets by ionosphere.

IV. REFERENCES

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