Propagation Through Dispersive and Nonlinear SMF Using Small Signal Analysis

Mihajlo Stefanović[1], Daniela Milović[2] and Aleksandra Mitić[3]

Abstract – **The small-signal analysis which takes into account both the lowest-order group velocity dispersion and the fiber nonlinearity influence on the signal propagation in a single-mode fiber is derived from the nonlinear Schrödinger equation. Fiber transfer functions are derived and used for optical pulse propagation thus avoiding split-step Fourier method.**

Keywords – **Small signal analysis, optical fiber dispersion, fiber nonlinearity, intensity modulation, optical propagation**.

I. INTRODUCTION

Data transmission in optical fiber communication systems is determined by a number of linear and nonlinear effects. The importance of both linear and nonlinear effects increases very rapidly with the capacity of the transmission systems. The joint action of the linear and nonlinear effects produces a great variety of optical phenomena in transmission systems. Many of these effects can easily distort the signal so that the data will be completely lost. Nevertheless, by an appropriate design of the system one can greatly reduce the detrimental consequences of these phenomena.

Today high-capacity systems are limited by a joint combination of dispersion and nonlinear effects. Dispersion-induced pulse broadening can be detrimental for optical communication systems. In the nonlinear regime, the combination of dipersion and nonlinearity can result in a qualitatively different behaviour. The anomalous dispersion regime is of considerable interest for the study of nonlinear effects because in this regime optical fibers can support solitons through a balance between the dispersive and nonlinear effects. The intensity dependence of the refractive index leads to a large number of interesting nonlinear effects. Self-phase modulation (SPM) refers to the self-induced phase shift experienced by an optical field during its propagation in optical fibers [1].

The impact of the interaction of dispersion and fiber nonlinearity on the fiber response was investigated numerically using the split-step Fourier method. The effects of the fiber nonlinearity and dispersion on the fiber transfer function (FTF) was found to be highly desirable to design properly long-haul optically amplified Gb/s transmission systems since it avoids the long computation time required by simulation and allows us to get more insight on the impact of the fiber nonlinearity on the FTF [2].

II. SMALL SIGNAL ANALYSIS

Signal propagation in a single mode fiber taking into account both group velocity dispersion (GVD) and the nonlinear Kerr effect (fiber nonlinearity) is described by the nonlinear Schrödinger equation (NLSE) [1]:

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\frac{j}{2}\;\beta_2\frac{\partial^2 A}{\partial t^2}=j\gamma|A|^2 A \tag{1}$$

where A is the slowly varying envelope amplitude, z is the longitudinal coordinate of the fiber and α is the power attenuation coefficient. The GVD is given by $\beta_2 = -\lambda^2 D/(2\pi c)$ with λ the carrier wavelength, D the dispersion parameter and c velocity of light in vaccum. γ is nonlinearity coefficient responsible for (SPM), given by $2\pi n_2/(\lambda A_{\text{eff}})$ with n_2 the nonlinear index coefficient and A_{eff} the effective core area.
The amplitude envelope can be written as

$$A(z,t)=\sqrt{p_T(z,t)}\cdot\exp\left[j\phi_T(z,t)\right] \tag{2}$$

where $p_T(z,t)$ and $f_T(z,t)$ are both real quantities denoting, respectively, the power and the phase of the optical field. If we replace the amplitude envelope in terms of power and phase derivatives and after some algebraic manipulations, the following set of two coupled nonlinear equation describing the power and phase of an optical field propagation through a nonlinear dispersive fiber valid for arbitrary input signal or noise is optained [3]:

$$\frac{\partial p_T}{\partial z}=\beta_2\left(\frac{\partial p_T}{\partial t}\cdot\frac{\partial \phi_T}{\partial t}+p_T\frac{\partial^2\phi_T}{\partial t^2}\right)-\alpha\cdot p_T \tag{3}$$

$$\frac{\partial \phi_T}{\partial t}=\gamma\cdot p_T-\frac{\beta_2}{8{p_T}^2}\left[2p_T\frac{\partial^2 p_T}{\partial t^2}-\left(\frac{\partial p_T}{\partial t}\right)^2-4{p_T}^2\left(\frac{\partial \phi_T}{\partial t}\right)^2\right] \tag{4}$$

To derive the small signal analysis we express the total power, $p_T(z,t)$ as the sum of the average power $P(z)$, and a noise or modulation term, $p(z,t)$ and the phase as the sum of the average phase $\Phi(z)$, and a noise or modulation term $\phi(z,t)$.

[1] Mihajlo Stefanovic is with the Faculty of Eletronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, e-mail:misa@elfak.ni.ac.yu
[2] Daniela Milovic is with the Faculty of Eletronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, e-mail:dacha@elfak.ni.ac.yu
[3] Aleksandra Mitic is with the Faculty of Eletronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia,e-mail:alekmi@elfak.ni.ac.yu

The small signal analysis implies that the average power is much larger than the noise or modulation term *P(z)>> p(z,t)* and the phase modulation or noise is small enough [4,5]. Substituting these components into the set of two coupled nonlinear equations for the power and phase of the optical field and neglecting the products of *p(z,t)* and *ϕ(z,t)* and their derivatives, a set of coupled linear differential equation governing the power and phase modulation terms is obtained. Using the normalized power $p_N(z,t)$ defined as $p(z,t)= p_N(z,t)\exp(-\alpha z)$ and doing Fourier transforms of both sides of the coupled linear differential equations governing the power and phase modulation terms, we obtain the set of linear differential equations [2,3]:

$$\frac{\partial \tilde{p}_N(z,\omega)}{\partial z} = -\beta_2\omega^2 P(0)\cdot\tilde{\phi}(z,\omega) \tag{5}$$

$$\frac{\partial \tilde{\phi}(z,\omega)}{\partial z} = \left[\frac{\beta_2\omega^2}{4P(0)} + \gamma\cdot\exp(-\alpha z)\right]\cdot\tilde{p}_N(z,\omega) \tag{6}$$

where $\tilde{p}_N(z,\omega)$ and $\tilde{\phi}(z,\omega)$ are the Fourier transforms of $p_N(z,t)$ and $\phi(z,t)$, respecitevely, *P(0)* is the average power at the fiber input and ω is the angular modulation frequency. Equation (5) and (6) govern the power and phase fluctuation evolution along the optical fiber taking into account GVD, nonlinear Kerr effect and fiber loss.

III. IM-IM CONVERSION FUNCTION

Since in IM-DD systems the information recovered only from the optical power amplitude, we focus on the IM solution. The information signal at the fiber input is carried by the IM term $\tilde{p}_{in}(\omega)$, while the transmitter chirp is included by the PM term $\tilde{\phi}_{in}(\omega)$.

For the purpose of theoretically analyzing frequency transfer function some conversion functions are given in [6]. If we assume that only intensity modulation is present and there is no phase modulation, we use IM-IM conversion function given in [6] as follows

$$C_{IM-IM} = \left.\frac{\tilde{p}_N(z,\omega)}{\tilde{p}(0,\omega)}\right|_{\tilde{\phi}(0,\omega)=0}$$

In case of a lossless and linear fiber, equations (5) and (6) can be solved analitically [5,7].

$$\tilde{p}(z,\omega) = \cos\left(\frac{\beta_2\omega^2}{2}\right)\cdot\tilde{p}(0,\omega) + 2P(0)\cdot\sin\left(\frac{\beta_2\omega^2}{2}\right)\cdot\tilde{\phi}(0,\omega) \tag{7}$$

$$\tilde{\phi}(z,\omega) = \cos\left(\frac{\beta_2\omega^2}{2}\right)\cdot\tilde{\phi}(0,\omega) - \frac{1}{2P(0)}\cdot\sin\left(\frac{\beta_2\omega^2}{2}\right)\cdot\tilde{p}(0,\omega) \tag{8}$$

we will now consider the propagation of optical pulse having Gaussian amplitude envelope

$$p(0,t) = \sqrt{P(0)}\exp(-(t/T_0)^2)$$

Optical pulse propagation is then simply simulated using equation (7) for the case of no phase modulation for typical standard single mode fiber (SMF) operating at wavelength of λ=1550 nm. Obtained results are shown graphically on Fig.1.

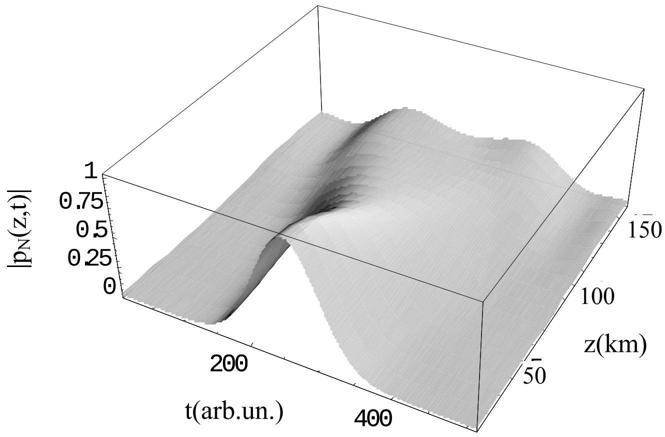

Fig.1. Spatial domain optical pulse propagation in linear dispersive regime (D=17 ps/nm/km)

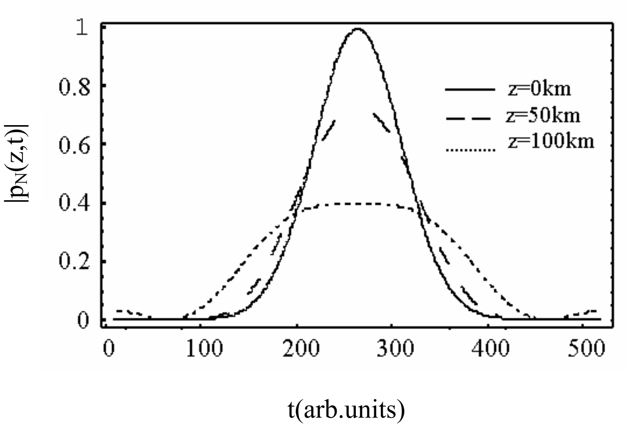

Fig.2. Pulse broadening due to dispersion for different propagation distances

When the optical power at the fiber input increases, nonlinear effects cannot be neglected and dispersive nonlinear propagation must be considered.
In case of a lossless fiber with the effects of fiber nonlinearity and dispersion, solving the equations (7) and (8) are obtained

$$\tilde{p}_N(z,\omega) = \cos(\xi z)\cdot\tilde{p}_N(0,\omega) + \frac{\beta_2\omega^2 P(0)}{\xi}\cdot\sin(\xi z)\cdot\tilde{\phi}(0,\omega) \tag{9}$$

$$\tilde{\phi}(z,\omega) = \cos(\xi z)\cdot\tilde{\phi}(0,\omega) - \frac{\xi}{(\beta_2\omega^2 P(0))}\cdot\sin(\xi z)\cdot\tilde{p}_N(0,\omega) \tag{10}$$

where $\xi = \sqrt{\left|K^2\right|}$ and $K^2=(\beta_2\omega^2/2)^2+\beta_2\omega^2\gamma P(0)$. If $K^2>0$ the solutions are stable. In the normal regime, $\beta_2>0$ and $K^2>0$, both the power and the phase have a stable solution. In the anomalous regime, $\beta_2<0$ and for modulation frequencies smaller than the angular critical frequency given by $\omega = \sqrt{4\gamma P(0)/\left|\beta_2\right|}$, $K^2<0$, power and phase present an unstable solution; otherwise solution is stable. A stable or unstable solution ($K^2>0$ or $K^2<0$, respecitively) depens critically on whether the GVD is normal or anomalus. The modulation instability is a phenomenon resulting from the iteration of dispersion and fiber nonlinearity. In the case of unstable solution the power or the phase modulation term will grow exponentially along the optical fiber [7].

For the case of no phase modulation, optical pulse propagation in spatial domain, for SMF parameters D=17 ps/nm/km and γ=1.05 $W^{-1}km^{-1}$, is shown on Fig.2. Inputu peak power value is P(0)=16dBm.

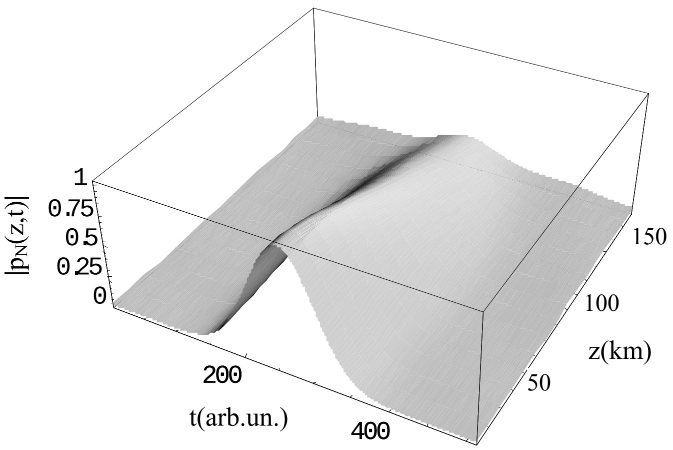

Fig.3. Optical pulse propagation in spatial domain through nonlinear dispersive SMF

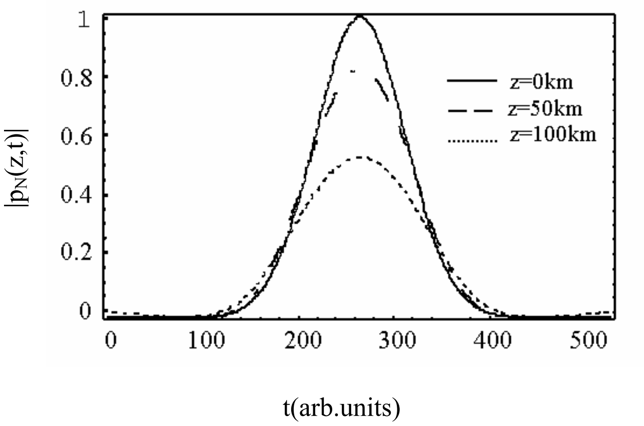

Fig.4. Pulse distortion due to dispersion and nonlinearity for different propagation distances

Equations (7) and (8) as well as equations (9) i (10) may be written by a matrix notation from which it may be seen that for any arbitrary phase and intensity modulation (or noise) at the fiber input, the corresponding phase and intensity modulation (or noise) at the fiber output, takes into account the correlation between the phase and intensity modulation and considers the second order dispersion effects and fiber nonlinearity, respectively. The conversion matrix is valid for any laser source, because it is derived without imposing any relation between phase and intensity of the field at the input of fiber, except the small signal assumption.

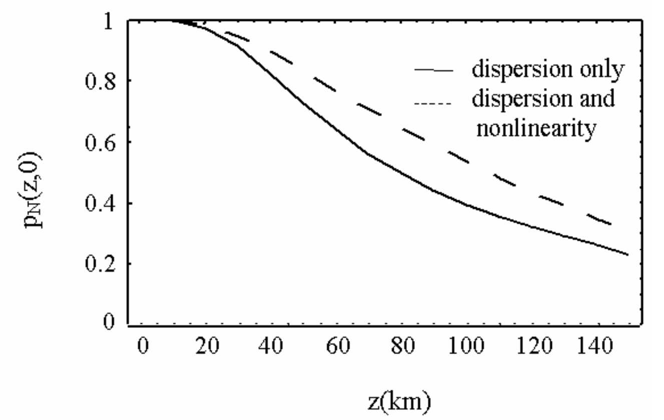

Fig.5. Power peaks versus propagation distance for: dispersion only (full line) and dispersion and nonlinearity (dashed line)

IV. CONCLUSION

Small signal analysis derived directly from nonlinear Schrodinger equation is a very usefull approximation that avoids split-step Fourier method and thus make the computing time less. Although it is restricted only to small input signals, this approximation is valid for any laser source. In this paper we considered optical pulse propagation for the cases of: 1) dispersion only and 2) dispersion and nonlinearity both included. It is shown that dispersion only causes more severe pulse distortion since nonlinearity compensate pulse broadening as well as pulse envelope degradation.

REFERENCES

[1] G. P. Agraval, *Nonlinear Fiber Opics*, Academic Press Inc., 2nd Edition San Diego, 1995.

[2] A.V.T. Cartaxo, B. Wedding and W. Idler, "Influece of Fiber Nonlinearity on the Phase Noise to Intensity Noise Conversion in Fiber Transmission: Theoretical and Experimental Analysis", *Journal of Lightwave Technology*, Vol. 16, Nº.7, July 1998.

[3] A.V.T. Cartaxo, B. Wedding and W. Idler, "Influence of Fiber Nonlinearity on the Fiber Transfer Function: Theoretical and Experimental Analysis", *Journal of Lightwave Technology*, Vol. 17, Nº 10, October 1999.

[4] A.V.T. Cartaxo and J. Morgano, "Rigorous Assessment of Small-Signal Analysis for Linear and Dispersive Optical Communication Systems Operating Near Zero-

Dispersion Wavelenght", *Journal of Lightwave Technology*, Vol. 17, Nº1, January 1999.

[5] J. Wang and K. Petermann, "Small Signal Analysis for Dispersive Optical Fiber Communication Systems", *Journal of Lightwave Technology*, Vol. 10, Nº 1, January 1992.

[6] F. Ramos and J. Marti, "RF Response of Analog Optical Links Employing Optical Phase Conjugation", *Journal of Lightwave Technology*, Vol. 19, Nº 6, June 2001.

[7] A. V. T. Cartaxo, "Small-signal Analysis for Nonlinear and Dispersive Optical Fibers, and its Application to Design of Dispersion Supported Transmission Systems with Optical Dispersion Compensation", *IEE Proc.-Optoelectron.*, Vol. 146, Nº 5, October 1999.