Analysis of Full Availability Loss System when the Input Stream is Peaked or Smooth

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Abstract: **This paper presents a generalized input Poisson stream, peaked or smooth. A teletraffic model for a full availability loss system with a generalized input stream is proposed. The idea is based on the analytic continuation of the Poisson distribution and the Erlang-B formula. We use techniques based on birth and death process and state-dependent arrival rates.**

Keywords: **Queueing theory, Performance evaluation, Teletraffic models.**

I. INTRODUCTION

The Poisson process is one of the simplest and most interesting stochastic processes [6,9]. One of the properties is that the mean and the variance are equal. These features simplify analysis, but introduce inaccuracy.

The traffic flows inside a network are not Poissonion in general [1]. For many real teletraffic systems the mean number of events in an interval is not equal to the variance. The offered streams are said to be peaked or smooth according to whether the variance is bigger or smaller than the mean value, respectively.

The Equivalent random theory (ERT) model is used to analyze overflow systems [5]. The Bernouilli-Poisson-Pascal (BPP) method is used to approximate the main congestion functions associated with peaked and smooth traffic in lost-call-cleared systems [3]. The BPP model represents peaked and smooth traffic by two separate models, and cannot represents arbitrary smooth traffic.

Network analysis really requires a technique that can represent any kind of traffic, peaked or smooth, within the same model [2,4]. All the methods are designed for a particular type of traffic, peaked or smooth, or, if they apply to both, do so using different models for different ranges of peakedness. The present method here meets the above requirements.

In this paper peaked and smooth input streams are defined. They will be called a generalized Poisson process. A calculation method for the time, call and traffic congestion probabilities and computation of the permissible offered traffic in a full availability loss system with a generalized Poisson input stream is presented.

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II. GENERALIZED POISSON PROCESS

The Poisson process is a pure birth process with an arrival rate λ independent of the system state. The stationary probability of having i customers in the system at time t are :

$$P_i(t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t} \tag{1}$$

Two more parameters, peakedness factor p and number of sources s, is introduced for the generalized Poisson process. Processes are said to be peaked, regular or smooth according to whether $p > 1$, $p = 1$ or $p < 1$, respectively.

The state probabilities $P_i(t)$ in the case of a generalized Poisson process are :

$$P_i(t) = \frac{\left[(\lambda t)^i / i!\right]^{1/p}}{\sum_{k=0}^{s} \left[(\lambda t)^k / k!\right]^{1/p}} \tag{2}$$

The mean value (the average number of arrivals in an interval of length t) is :

$$M(t) = \sum_{i=1}^{s} i\, P_i \tag{3}$$

The variance of the number of arrivals in an interval of length t is :

$$V(t) = \sum_{i=0}^{s} [i - M(t)]^2 \, P_i(t) \tag{4}$$

when $p = 1$, $M(t) = \lambda t$ and $V(t) = \lambda t$ i.e. it is a regular Poisson process.

III. LOSS SYSTEM WITH N-SERVERS

Let us consider a full availability group of size n, number of sources $s > n$ and a generalized Poisson input stream. This is a birth and death process and we can use the general solution as given in [7]:

$$P_k = \frac{\prod_{i=0}^{k-1} \lambda_i / \mu_{i+1}}{1 + \sum_{v=1}^{n} \prod_{i=0}^{v-1} \lambda_i / \mu_{i+1}} \qquad k = 0,1,2,\ldots,n \tag{5}$$

The blocked calls cleared system may be described by selecting the birth-death coefficient as follows :

$$\begin{aligned} &\lambda_k = \lambda^{1/p} (k+1)^{1-1/p} && k = 0,1,2,\ldots,n \\ &\lambda_k = 0 && k > n \\ &\mu_k = k \ \mu^{1/p} && k = 1,2,3,\ldots,n \end{aligned} \tag{6}$$

The arrival rate is state-dependent and both arrival and service rate depends from the peakedness factor p. This loss system is always ergodic. The finite state-transition diagram is shown in Figure 1.

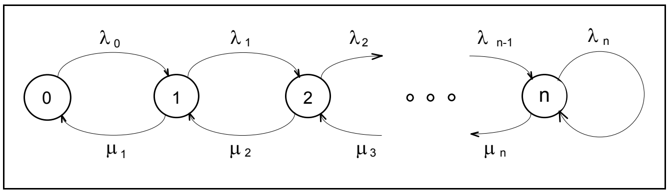

Fig. 1.

Applying these coefficients to the general solution of the birth and death process and using traffic intensity $a = \lambda/\mu$ we obtain the generalize Erlang distribution

$$P_k = \frac{(a^k / k!)^{1/p}}{\sum_{i=0}^{n} (a^i / i!)^{1/p}} \qquad k = 0,1,2,...,n \tag{7}$$

When the peakedness factor $p = 1$, we get the Erlang distribution.

The offered traffic is calculated by means of the average arrival rate and the mean holding time

$$A = \frac{1}{\mu^{1/p}} \sum_{k=0}^{n} \lambda_k P_k = a^{1/p} \sum_{k=0}^{n} (k+1)^{1-1/p} P_k \tag{8}$$

The carried traffic is the expectation of the number of calls existing in the steady state

$$A_c = \sum_{k=0}^{n} k\, P_k \tag{9}$$

Assume that the number of the servers is equal to the number of the sources $n = s$. In this case the whole offered traffic is carried and it is called the intended traffic load. The intended traffic is the equilibrium number of busy servers

$$A_i = \sum_{k=1}^{s} k\, P_k \tag{10}$$

The variance of the intended traffic is

$$V(A_i) = \sum_{k=0}^{s} (k - A_i)^2 P_k \tag{11}$$

The peakedness of the intended traffic is the variance to mean ratio

$$z = \frac{V(A_i)}{A_i} \tag{12}$$

IV. CONGESTION PROBABILITY

The time congestion probability B_t describes the fraction of time that all n servers are busy

$$B_t(a,p,n) = P_n \tag{13}$$

The call congestion probability B_c is the ratio of the number of calls lost to the number of calls offered, during a certain period in statistical equilibrium

$$B_c(a,p,n) = \frac{\lambda_n P_n}{\sum_{k=0}^{n} \lambda_k P_k} = \frac{(a^n / n!)^{1/p} (n+1)^{1-1/p}}{\sum_{k=0}^{n} (a^k / k!)^{1/p} (k+1)^{1-1/p}} \tag{14}$$

The traffic congestion probability B_a is the ratio of traffic lost and intended traffic

$$B_a(a,p,n) = \frac{A_i - A_c}{A_i} \tag{15}$$

V. CALCULATION OF THE LOSS PROBABILITY

The traffic intensity a is not equal to the intended traffic in a case of a generalized Erlang process because, we calculate the power of the Erlang unsymmetrical distribution. That is why we have to calculate the intended traffic A_i and the peakedness z when defining the traffic intensity a and peakedness factor p.

From the practical point of view we first define the intended traffic A_i and the peakedness z and after that calculate the traffic intensity a and peakedness factor p.

A fundamental question about the system defined by Eqs. (7), (10) and (12) is whether there exist solutions a, p for an arbitrary A_i, z. Although no formal proof seems to exist, this seems to be the case and the solution seems to be unique.

We can find solutions of the above system with the iterating method of consecutive replacements. This method is not applicable when the intended traffic per server $A_i /n < 0.25$ *erl* end $z > 1$ because the relative difference between the traffic intensity a and A_i is bigger. In other words it is difficult to find a and p when the intended traffic is small and peaked.

The time congestion obeys the interesting recurrence which is a generalization of well-known recurrence for the Erlang-B function

$$B_t(a,p,n) = \frac{B_t(a,p,n-1)}{(n/a)^{1/p} + B_t(a,p,n-1)} \tag{16}$$

where the initial value $B_t(a,p,0) = 1$.

In this paper is used another method [8], which is convenient for numerical computation. The time congestion probability is exactly expressed as a sum

$$\frac{1}{B_t(a,p,n)} = \sum_{i=0}^{n} T_i \tag{17}$$

where the terms T_i are obtained using the following recurrence with $T_o = 1$

$$T_i = T_{i-1} \left(\frac{n-i+1}{a} \right)^{1/p} \tag{18}$$

The call congestion probability is exactly expressed as a sum

$$\frac{1}{B_c(a,p,n)} = \sum_{k=0}^{n} U_k \tag{19}$$

where the terms U_k are obtained using the following recurrence

$$U_k = U_{k-1}\left(\frac{n-k+1}{a}\right)^{1/p}\left(\frac{n-k+1}{n-k+2}\right)^{1-1/p} \tag{20}$$

with $U_o = 1$.

VI. CALCULATION OF THE TRAFFIC INTENSITY

Computation of the permissible offered traffic for a given number of servers, sources, peakedness and given value of loss probability is frequently required in the practical design of communication systems.

We use the Newton's iterative method for computation of a simple zero of nonlinear equation

$$\Psi(a) = 0 \tag{21}$$

An appropriate function is

$$\Psi(a) = a[B_t(a) - \beta_t] \tag{22}$$

where β_t is given time congestion probability.

The first derivative of this function with respect to a is

$$\Psi(a) = B_t(a) - \beta_t + \frac{B_t(a)\,(n - A_c)}{p} \tag{23}$$

The iterations are defined by

$$a_{r+1} = a_r - a_r \frac{B_t(a_r) - \beta_t}{B_t(a_r) - \beta_t + \dfrac{B_t(a_r)}{p(n-A_c)}} \tag{24}$$

The starting value is

$$a_o = n/(1-\beta_t) \tag{25}$$

VII. NUMERICAL RESULTS

In this section we give numerical results obtain by a Pascal program on an IBM PC. The described methods were tested on a computer over a wide range of arguments.

Figure 2 shows the generalized Poisson distribution where the intended traffic is $A_i = 10$ *erl*, the number of the sources $s = 100$ and the peakedness z is change from 0.25 to 4. It will be seen that when the peakedness z increase the probability distribution becomes broad about the mean.

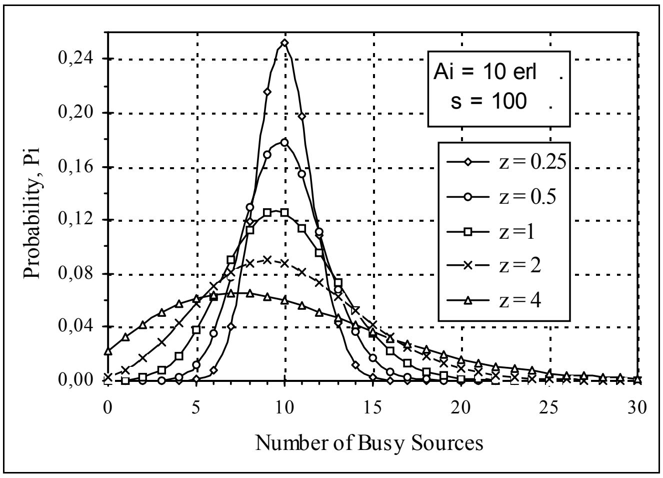

Fig. 2

Figure 3 illustrates the time congestion probabilities in a full availability loss system with 20 servers and 100 sources as functions of the intended traffic per server in a generalized input Poisson stream. The case with $z = 1$ and $s = \infty$ corresponds to the Erlang-B formula. The peakedness has the same value as in Fig. 2.

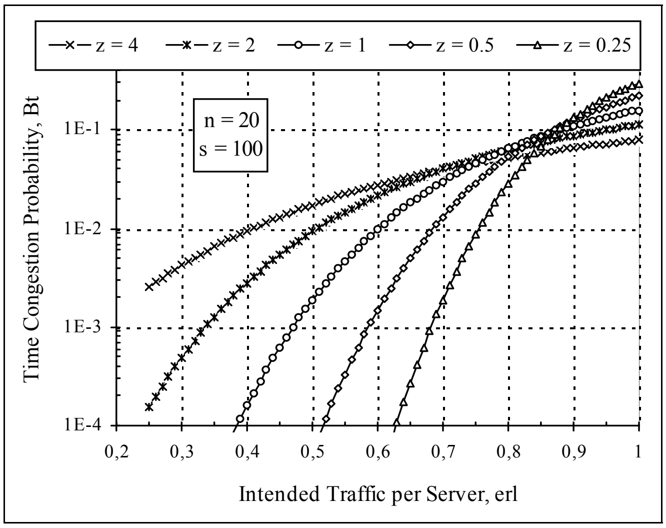

Fig. 3

It will be seen that when the intended traffic per server is from 0*.8 to 0.9 erl* the peakedness influence of the time congestion probability is negligibly.

When the intended traffic per server is less than *0.8 erl* the peaked stream increase and the smooth stream decrease the time congestion probability. When the intended traffic per server is greater then *0.9 erl* the time congestion probability is less in the peaked stream case than in the smooth one.

Figure 4 shows the time, call and traffic congestion probabilities in a full availability loss system with 20 servers and 100 sources as functions of the intended traffic per server in a generalized input Poisson stream. The peakedness z has the value 0.5, 1 end 2.

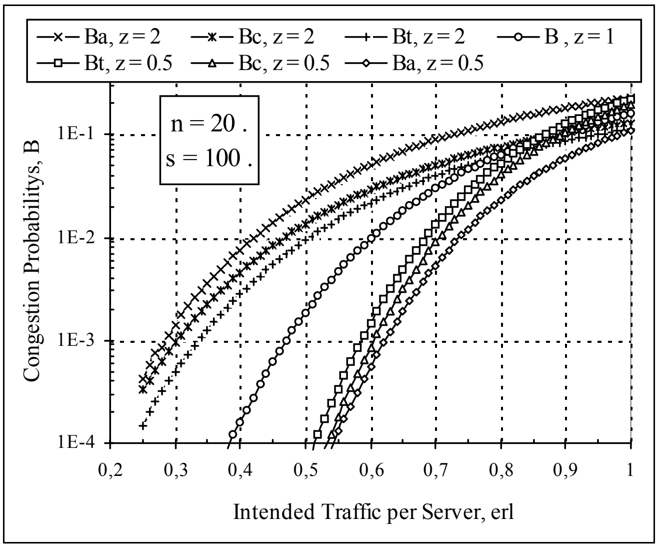

Fig. 4

Figure 5 is the traffic load curve. It will be seen that the smooth input stream increase and peaked input stream decrease the permissible offered traffic.

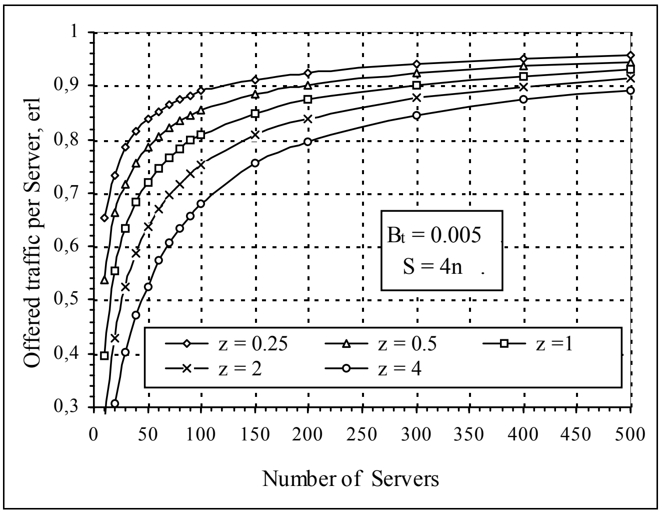

Fig. 5

There are differences between the present results and the congestion functions calculated by the Equivalent random theory (ERT) end by the Bernoulli-Poisson-Pascal (BPP) methods. The ERT overflow model uses an equivalent traffic as a generator of overflow traffic which is a part of the Poisson distribution. The BPP model is a state dependent arrival process with linear arrival rates.

Wallstrom [10] has derived the Binominal moments of the overflow traffic from a finite trunk group, when the offered traffic process is an arbitrary state-dependent Poisson process. This is applicable to the proposed model.

In the following we compare the teletraffic model for a full availability loss system when the input stream is a generalized Poisson process with the ERT and BPP models.

VIII. CONCLUSIONS

In this paper a generalized Poisson process is defined. A basic model for a full availability loss system in a generalized input Poisson stream is introduced and explained.

The proposed method provides a unified framework to model peaked and smooth traffic. Numerical results and subsequent experience has shown that this method is accurate and useful in both analyses and simulations of traffic systems.

The importance of a full availability loss system in a case of a generalized Poisson input stream comes from its ability to describe behavior that is to be found in more complex real queueing systems. It is the case in a general traffic system, which is an important feature in designing telecommunication systems.

The advantages of simplicity and uniformity in representing both peaked and smooth traffics make this model attractive for modelling traffic in network analysis and synthesis.

With the proposed model it is simple to compute the distribution parameters, and it has excellent accuracy from a numerical point of view.

In conclusion, we believe that the presented formulae will be useful in practice.

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