The Visualization of the Electromagnetic Field in Microwave Applicators

Vladan D. Stanković, Aleksandar Lj. Stošić, Dalibor Ž. Ranđelović

Abstract - **These papers present field patterns inside an microwave cavity. Routines wrote in MATLAB were used for the visualization of the field patterns. Heating loaded microwave cavity were considered at the end of these papers.**

Keywords- **Microwave applicators, cavity**

I. INTRODUCTION

Users of microwave cavities can be unfamiliar with some of the salient features of their equipment. Such applicators may be, for example, a domestic microwave oven at home for cooking food, or a specialized piece of microwave equipment in a laboratory for research purpose. The main part of microwave aplicators is cavity (the empty space surrounded by metallic walls where the material is placed) such as where the low-and high-intensity regions exist. Without the aid of computer simulation substantiated with reliable measurement, it is difficult to build a mental picture of what is actually going on inside the loaded cavity.

A good start in understanding microwave heating cavities is to consider the field patterns, referred to as modes, inside an empty microwave cavity. A knowledge of the modal field distribution leads to an understanding of the power loss density (i.e., hot and cold areas) inside the load. These papers concentrates on the field.

It is undeniable that the multimode cavity in practice always contains a load, and to some extent the study of the empty cavity can be considered irrelevant. However, much can be derived from the examination. Moreover, it also serves as a good background for understanding the waveguide-fed loaded cavity situation.

II. FIELD EQUATIONS

With Maxwell's equations aplied to a metal-walled rectangular cavity subject to the appropriate boundary conditions at each wall, the field equations are separated into TE_{mnp} (H_{mnp}) and TM_{mnp} (E_{mnp}) modes. The modes' spatial field distributions are determined by the size of the cavity. With the *z*-axis as the propagation direction, TE modes have no electric field component along that axis. TM modes have no magnetic field component along the same axis. Note that this nomenclature is typical but not unique.

The derivation of the field components is found in many textbooks. The equations follow that of a rectangular waveguide with additional boundary conditions provided by two plates shorting both ends of the guide. The field components for TE_{mnp} modes are given by

$$E_x = \left(\frac{j\omega\mu_o}{h^2}\right)\left(\frac{n\pi}{b}\right)H_o \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right) \quad (1)$$

$$E_y = -\left(\frac{j\omega\mu_o}{h^2}\right)\left(\frac{m\pi}{a}\right)H_o \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right) \quad (2)$$

$$E_z = 0 \quad (3)$$

$$H_x = -\left(\frac{1}{h^2}\right)\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)H_o \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right) \quad (4)$$

$$H_y = -\left(\frac{1}{h^2}\right)\left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)H_o \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right) \quad (5)$$

$$H_z = H_o \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right) \quad (6)$$

For TM_{mnp} modes,

$$E_x = -\left(\frac{1}{h^2}\right)\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)E_o \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right) \quad (7)$$

$$E_y = -\left(\frac{1}{h^2}\right)\left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)E_o \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right) \quad (8)$$

$$E_z = E_o \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right) \quad (9)$$

$$H_x = \left(\frac{j\omega\varepsilon_o\varepsilon}{h^2}\right)\left(\frac{n\pi}{b}\right)E_o \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right) \quad (10)$$

$$H_y = -\left(\frac{j\omega\varepsilon_o\varepsilon}{h^2}\right)\left(\frac{m\pi}{a}\right)E_o \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right) \quad (11)$$

$$H_z = 0 \quad (12)$$

where

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad (13)$$

m, *n,* and *p* are the integer numbers of half-sinusoidal variations of the field along the principal coordinate axes, and *a*, *b,* and *d* are the cavity dimensions along the *x*, *y,* and *z* coordinates, respectively.

From either the TE or TM mode equations, it is apparent that the electric and magnetic fields are spatially 90 degrees out of phase. The modes in the cavity are represented by a resonant circuit where the stored energy changes from electric to magnetic fields and back. During each cycle, as the magnetic field decreases, the electric field increases and vice versa. The energy is stored volumetrically unless losses are included. Losses are due to the metal walls and the load, if included.

To help in understanding the mathematics involved, an analogy can be used whereby an elastic string attached to two opposite fixed walls is plucked. The walls are boundary conditions. If the string is plucked in the middle, it will generate a standing wave pattern in one dimension. If three strings are attached in three orthogonal planes, then plucking all of them at the same time could hypothetically generate a three-dimensional standing wave pattern. This pattern can be viewed in the same light as the field in a multimode cavity.

III. Theoretical Mdes Inside the Empty Cavity

With the cavity several wavelengths long in at least two dimension in a given frequency range, a number of resonant modes are supported. The dimensions determine the number of half-wavelengths in each of the principal directions. For an empty cavity, each of these modes exhibits a sharp resonant response. The number of modes that exist is given by

$$\omega_{mnp}^2 \mu_o \varepsilon_o \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \qquad (14)$$

In an empty cavity, $\varepsilon = 1$. For a cubic structure, $a = b = d$, (14) becomes

$$4\left(\frac{d}{\lambda}\right)^2 = m^2 + n^2 + p^2 \qquad (15)$$

To dwell briefly on details, certain modes do not exist. For TM modes, only p can be zero (i.e., TM_{0np} and TM_{m0p} cannot exist). For TE modes, only p cannot be zero (i.e., TE_{mn0} cannot exist). The nonexistence of these modes can be checked by substituting the zero index in the electric and magnetic field equations. TE_{00p} also cannot exist. In this case, the electric field components are zero but one of the magnetic field components, the one along the z axis, is not zero. Since the magnetic and electric fields have to exist simultaneously, this mode therefore cannot occur in the cavity. Modes with nonzero indices are degenerate (i.e., they exist as TE and TM with the same frequency). The degenerate modes have the same mode indices but the field patterns are different. For example, appling (14) on noncubic cavity, wich dimension is $360 \times 350 \times 260 mm$, for mode 1-4-3 the calculated frequency is 2.4697 GHz, and it is also degenerate. Modes that contain a zero index exist as either TE or TM, excluding those that do not satisfy the boundary conditions stated above. For a *cubic* cavity, besides TE and TM being degenerate, any mode with a permutation of the same three indices is also degenerate.

Note that (14) applies to an empty or a fully loaded cavity; it does not apply to a partially loaded cavity. Usually, only part of the cavity is filled and therefore more rigorous methods (i.e., simulation) than this simple analytic expression are used.

IV. Noncubic Cavity

Althought most microwave cavities in use are noncubic, cubic cavities can also be used. At 2,45 GHz, the maximum number of degenerate modes for all cubic cavities with a volume less than or equal to 100L is 18. The cubic cavity is not as popular as the noncubic one because it produces too many degenerate modes. For example, at 2.4288 GHz there are 12 modes, and at 2.4535 GHz there are 21 modes. A reason for not choosing a cubic cavity is that if the source frequency is shifted, there is the possibility that all the modes might be missed altogether. However, this does not take into account the effect of the load, the feed coupling that sometimes splits the modes, and the feed positioning that will not excite all the degenerate modes.

Figure 1 shows a noncubic rectangular multimode cavity used here to study field patterns. The dimensions of the cavity are $360 \times 350 \times 260$ mm. The cavity has four entry ports on one wall. The coupling between these ports when they are excited simultaneously is very important. Entry ports that are not used are closed off with carefully made plates. One feed port, slightly off-center, is placed on the opposite wall. One of the walls is removable so that different metal plates with different feed positioning can be tested. The plate is firmly clamped to the rest of the cavity using winged nuts.

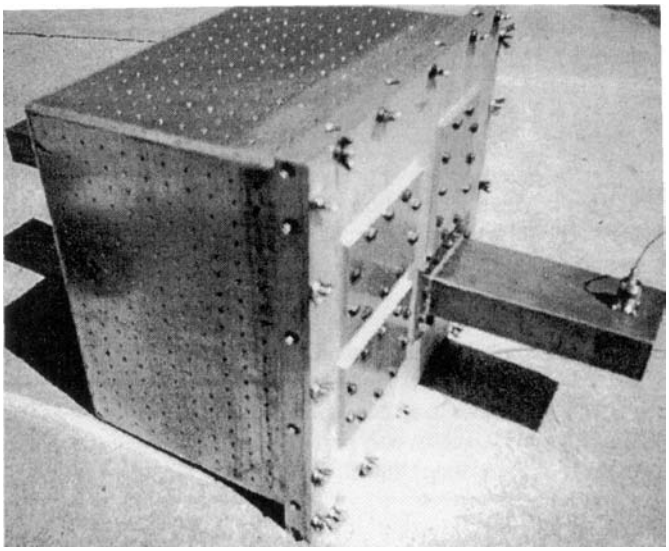


Figure 1. *Multimode cavity with multiple entry ports and arrayed holes for electric field probing.*

The excitation is achieved through a TE_{10} mode WR340 waveguide with dimension $86 \times 43 \times 200$ mm. The cavity is made of aluminum ($\sigma = 3.54e7 S/m$) with thicknesses of 6 mm on the wall with the feeding ports and 4,5 mm on the remaining walls. The whole box is seam welded. Holes of 3 mm in diameter are drilled on three of the faces. The holes are

spaced at a distance of 20 mm from each other and are used in the probing method to detect the electric field.

IV. Q OF A CAVITY

The quality factor of a cavity at resonance is defined as

$$Q = 2\pi \frac{Energy\ stored}{Energy\ lost\ per\ cycle} \tag{16}$$

If a measurement of S_{11} with frequency is available , the Q of a cavity is found from

$$Q = \frac{f_o}{\Delta f} \tag{17}$$

where f_o is the resonant frequency and Δf is the half-power frequency bandwith.

A cavity, in practice, has walls with finite conductivity. This leads to losses and a reduction in stored energy. The power supplied is also dissipated in the door seal, the mode stirrer, and the feed. With the presence of a load inside the cavity, the total dielectrically loaded cavity Q, Q_{Loaded} is given by

$$\frac{1}{Q_{Loaded}} = \frac{Energy\ lost\ per\ cycle\ inside\ the\ cavity}{2\pi\ Energy\ Stored} + \frac{Energy\ lost\ per\ cycle\ inside\ the\ load}{2\pi\ Energy\ Stored} \tag{18}$$

If the load is a good absorber of microwave energy, then the Q_{Loaded} decreases to such an extent that the loaded system can almost be thought of as nonresonant.

V. Fields in an Empty Rectangular Non Fed Resonant Multimode Cavity

Mathematical explanations of modes are widely available. These papers adds a pictorial view of higher-order modes through simulation. The visualization is useful as it gives a clear display of the modes and their corresponding field magnitudes and directions. This is important for feed positioning and feed crosscoupling studies. The modes are checked analytically using a MATLAB routine, and experimentally. The analytical expression is useful as it gives a quick and informative means of examining the field behavior.

A pictorial set of cavity TE mode 1-4-3 and its corresponding field distribution is presented in Figures 2,3,4, and 5. These pictures are ploted in MATLAB. The model in Figures 5a and 5b showing the arrow plots is sectioned at a distance of 5 mm from the surface of the cavity. As only dielectric material that absorbs and is heated by the electric field is considered, electric field plots are presented. However, there are instances where consideration of the magnetic field pattern is essential, manly in heating ferromagnetic materials. For interest, therefore, one magnetic field arrow plot is shown (Figure 5b).

In Figure 5a, the electric field is terminated normally on the walls. The tangential field cannot exist along the walls in the *z*-axis as this field must be zero. The direction of the field is 90 degrees to the axis of propagation, making it a TE mode. The magnitude of the field at a point is depicted either by its density or by the lenght of the directed lines in the vicinity of the point. The red shading therefore corresponds to the maximum electric field. Note that two quarter variations along the *x*-axis in Figure 4 equals the one half-sinusoidal variation. For higher-order modes, the electric field can form loops surrounding a changing magnetic field in the middle of the cavity. Figure 5b has their magnetic field 90 degrees to the propagation axis, making these modes transverse magnetic. Therefore, the magnetic fields form loops around the changing electric field to satisfy Maxwell's equations.

The cavity is rectangular and therefore contains lines of field symmetry. There is no difference in placing a feeding TE_{10} waveguide on one wall or the wall facing it. Each walls has two axes of symmetry.

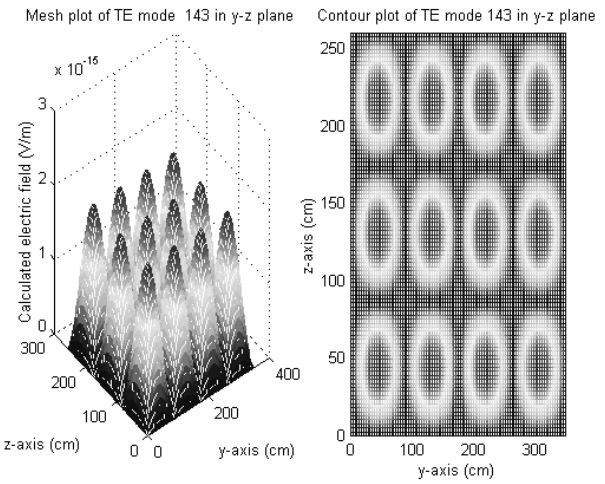


Figure 2. *Electric field plot of TE mode 1-4-3 in the y-z plane*

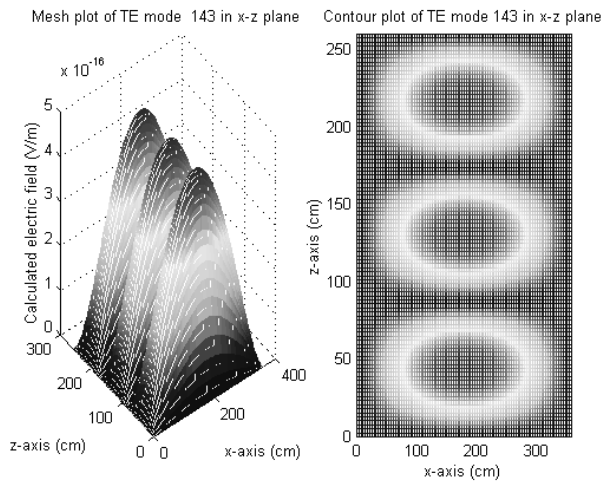


Figure 3. *Electric field plot of TE mode 1-4-3 in the x-z plane*

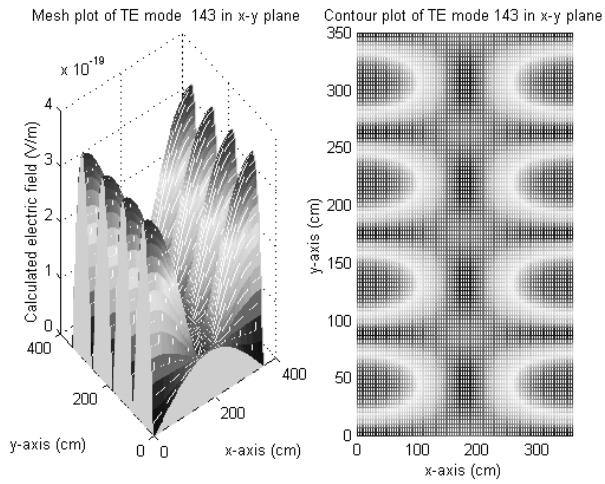
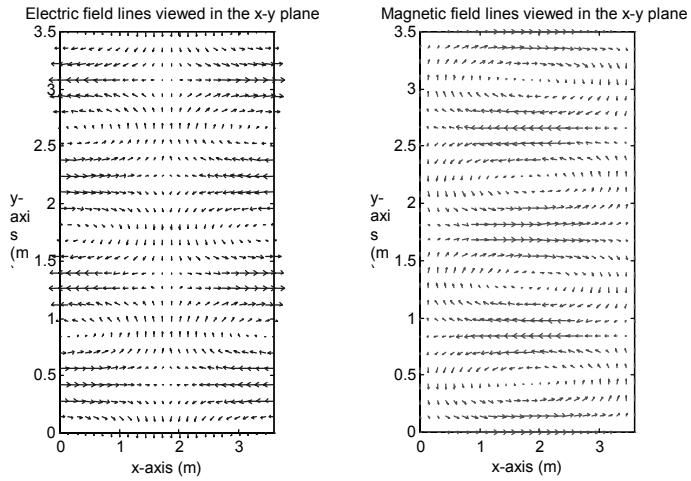


Figure 4. *Electric field plot of TE mode 1-4-3 in the x -y plane*



(a) (b)

Figure 5. *Electric (a); magnetic (b); field arrow plots TE, TM mode 1-4-3 in the x-y plate at* $z = 5mm$.

VI. CONCLUSIONS

Ever since microwave heating was discovered, one of the issues that has drawn much interest is the heating uniformity in a load. The results indicate that the load significantly influences the pattern inside the cavity with patchy regions of minimum and maximum electric field. The electric field patterns, inside loaded cavity, cannot be correlated with those present inside an empty cavity. Uneven field distribution creates the socalled hot and cold spots. Hot spots could, for example, contribute to the phenomenon of thermal runaway typical of ceramic materials. For food engineers, cold spots are unwelcome as they allow bacteria to thrive if the temperature is not sufficiently high enough to kill them, wich could cause food poisoning. Therefore, a more uniform heating is generally desirable. If a better distribution is the objective, then the fallowing questions are sensibly asked:

1. Where is a good position inside the cavity?
2. What size and shape load should be used?
3. What size and shape cavity should be used? (Some researchers use an octagonally shaped cavity instead of the usual rectangular cavity).
4. Where should the feeds be placed and how many are needed?
5. How broad is the source spectrum of each magnetron and at which frequency is each spectrum centered? (Each one influences the number of modes and hence the heating pattern inside the cavity).
6. How easy is it to position the other feeds for low crosscoupling?

All these factors will have a bearing on the field pattern. A further aspects is that the dielectric properties of most materials change with temperature. Because of the unpredictable field pattern inside the loaded cavity, one can interpret the results confidently provided the simulation is supported with reliable experimental agreement.

Hardware and software are very powerful nowdays, so its are good tools for simulations and visualizations a field pattern inside loaded cavity.

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