

Computer Processing Of 1-D Medical Signals With Incomplete Spectrum

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Abstract - The problem of removing the noise, concentrated in a determined frequency band on a physical device for receiving a signal with a finite Fourier spectrum can be solved by means of interpolation of the spectrum in the frequency band, if out of this band the spectral components are known.

The results from the restoration of 1D medical signals, saved on the hard disk on the computer in format Wave sound (*.wav) are presented in the paper. In the computing experiments the medical signal loss is simulated by removing the spectral values and the signal is restored by interpolation, using a mathematical method for analytical continuation of finite functions. The programs are written in Fortran Power Station, and the visualization of the results is made by means of Excel. The algorithm of the computing experiment is described. The error of the interpolation is computed. It is possible to hear the sound files— from the signal with incomplete spectrum and from the restored signal.

The method for analytical continuation of the functions, described in the paper can be applied both to interpolation and to extrapolation of signals with any kind of dimension with incomplete Fourier spectrum in communication, medicine etc.

I. MATHEMATICAL METHOD USED FOR SOLVING THE PROBLEM OF RESTORATION OF 1-D MEDICAL SIGNALS WITH INCOMPLETE SPECTRUM

The files with extension WAV are Windows standard 1-D sound files. They can be heard with every WAV – player – for example Sound

Recorder or Media Player. Every WAV file begins with file header (44 Bytes) followed by the samples of type Word.

These sound files are one-dimensional finite signals. They are presented as functions of the time in the positive direction of the axis t , but the signals spectrums are functions of the frequency in the finite interval $[0, \alpha]$. They enter in the

Wiener's class of whole functions W_{α}^{+} [1,2]. When the spectrum is lost in the definite narrow frequency band it is possible to restore it by means of formulas for interpolation of analytical functions. This possibility is realized with the Aizenberg's formulas [1] for analytical continuation of finite functions from the Hardy's and Wiener's spaces. In short the method of Aizenberg consists in [1]:

If N_j is a limited sequence of different points in the halfplane $\{z_j \in D: \text{Im } z_j > -\sigma\}$ which has no points of density over its contour $1 \leq j \leq n$, let us consider the following problem Dimitar

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of restoration of function $f(z)$ in D_{σ}

where $D_{\sigma} = \bigcap_{j=1}^n \{z_j \in D: \text{Im } z_j > -\sigma\}$, in the set

$M = N_1 \times N_2 \times \dots \times N_n$. For $N_l = \{x_{lj}\}$ is valid (1):

$$\omega(m, u, p, l) = \frac{x_{lp} - \bar{x}_{lp} + 2i\sigma}{u - \bar{x}_{lp} + 2i\sigma} \prod_{\substack{j=1 \\ j \neq p}}^m \frac{(u - x_{lj})(x_{lp} - \bar{x}_{lj} + 2i\sigma)}{(u - \bar{x}_{lj} + 2i\sigma)(x_{lp} - x_{lj})} \quad (1)$$

For $f \in H^2(D_{\sigma})$, $z \in H^2(D_{\sigma})$ is valid:

$$f(z) = \lim_{m \rightarrow \infty} \sum_{k_1=1}^m \sum_{k_2=1}^m \dots \sum_{k_n=1}^m f(x_k) \prod_{l=1}^m \omega(m, z_l, k_l, l) \quad (2)$$

$$x_k = (x_{1k_1}, \dots, x_{nk_n}), k = (k_1, \dots, k_n)$$

where H is a Hardy's class of functions, $\sigma > 0$ is a parameter, \bar{x} is a conjugate complex value of x .

The following formula can be used for interpolation of function $f(x)$ in Wiener class W_{α}^{+} :

$$f(x) = \lim_{k \rightarrow \infty} \sum_{k=1}^m f(x_k) \frac{2i\sigma}{x - x_k + 2i\sigma} \prod_{\substack{j=1 \\ j \neq k}}^m \frac{(x - x_j)(x_k - x_j + 2i\sigma)}{(x - x_j + 2i\sigma)(x_k - x_j)} \quad (3)$$

2. Experiments on the computer processing of 1-D sound signals with incomplete spectrum.

The loss of a part of the signal spectrum is modeled by removing a part of the spectral values, when they are replaced with zero. A noise is heard during the hearing of the sound signal – the signal's quality is lowered. After execution of the interpolation algorithm the quality is improved – the noise is removed. For example a standard sound Windows file is used – badaddr.wav, which has 4971 values. The subroutine realizing the algorithm for the restoration of the signal, using (3) is written in Fortran Power Station v.4.0. The steps of the algorithm of the computing experiment are:

1. Removing the file header.
2. Finding the spectrum of the signal
3. Simulation of signal spectrum loss.
4. Transformation in the time area, adding the file header and hearing the sound file.
5. Restoration of the spectrum, found in Step 3.
6. Transformation in the time area, adding the file header and hearing the sound file.
7. Visualization and additional estimation of the received results.

Finding the signal spectrum by Fast Fourier Transformation (FFT) (Step 2) is realized with double precision by the subroutine FFT3D from the mathematical

library IMSL. The higher precision in the computation leads to a smaller error in signal restoration. The same subroutine FFT3D is used for finding the signal values in Step 4.

The analytical continuation of the spectrum (Step 5) is computed by a subroutine, applying (3) as the number m of the known spectral values and the parameter σ are defined by searching of the minimal root mean-square error ε by formula (4) between the real and interpolated values:

$$\varepsilon = \sqrt{\frac{1}{n} \sum_{i=0}^n (x_i - \tilde{x}_i)^2} \quad (4)$$

where x_i and \tilde{x}_i are the values of the real and restored sound signal.

After a determined value of m , the increasing of restored frequencies interval comes to increasing of root mean-square error. The absolute error between the absolute value of the approximation and real spectral function is also computed.

Experiments have been made for restoration of 1, 2, 3 lost spectral values.

The least error is calculated for $m=23$ (absolute error $\varepsilon_{abs} = 0,1 \cdot 10^3$ and mean-square error $\varepsilon_{sq} = 0,17 \cdot 10^4$) for loss of 1 frequency (under number 13) of the spectrum signal. Another estimation (Step 7) of the received results can be made, using Student and Fisher criteria [3]. The visualization of the signals can be realized with Matlab and as in the particular case with Excel v.97, because the number of the values is less than 65536. The "hearing" of the results is the simplest and universal method for quality estimation in restoration of the sound signals.

The original sound file is presented on Figure 1:

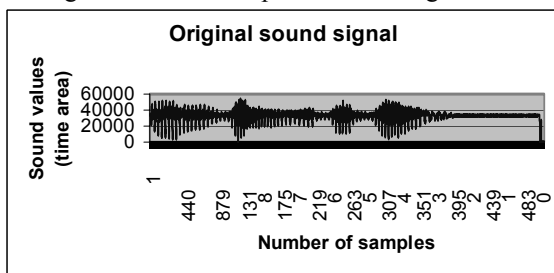


Figure 1. Original sound signal

On Figure 2 a graphic of the signal with incomplete spectrum is presented where 1 spectral line loss. The visualization is made in the time area (Step 4 from the algorithm, described above).

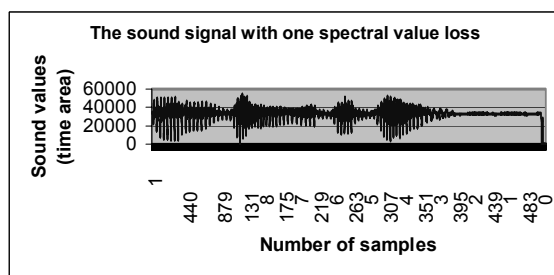


Figure 2. The sound signal with one spectral value loss

On Figure 3 a graphic of the interpolated signal is presented where the restoration is made after 1 spectral line loss (after Step 6).

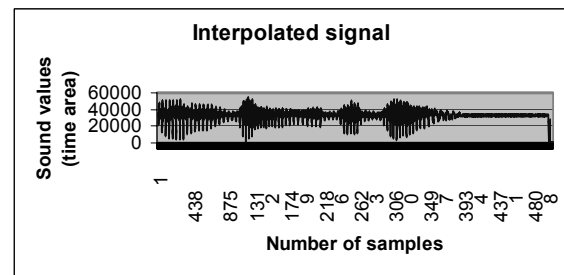


Figure 3. Interpolated sound signal

What is the place of computer processing of sound signals with incomplete spectrum in the educational process?

The specialists are acquainted with the theoretical basis of the problem for analytical continuation of the finite functions and the used mathematical method of Aizenberg in the case of practical application. They follow the algorithm described above, using the developed programs in Fortran Power Station. They can execute the program with various values of m and different interval of spectrum loss in the interactive mode. The output of the program execution includes files with the signal's values in the spectral and time area and the errors' values. The visualization of the signals can be made by the students themselves in Excel as presented on Figure 1, Figure 2, Figure 3 and their corresponding graphics in the frequency area.

III. CONCLUSIONS

The next conclusions can be made from the described experiments:

- the method used for continuation of analytical functions leads to restoration of the damaged sound signals;
- when the spectrum is lost in the greater continual interval (more than 2 spectral values, placed next to each other) the mean square error grows and the application of the method is not right;
- the best results can be reached when the method is used for restoration of the received spectral values from the apparatus – i.e. in communications etc.

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