Nondeterministic (3,2)-Semigroup Automata

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Abstract – **The aim of this paper is to define a noneterministic (3,2)-semigroup automaton and to give them conection with deterministic (3,2)-semigroup automaton**

Keywords - **(3,2)-semigroup, (3,2)-semigroup automaton, (3,2)-language, nondeterministic (3,2)-semigroup automaton**

I. INTRODUCTION

Our goal in writing this talk is to examine a nondeterministic (3,2)-semigroup automaton and to proof that they can not always be converted into a deterministic one. In that means, we are given an example.

II. (3,2)-SEMIGROUP AND (3,2)-SEMIGROUP AUTOMATON

Here we recall the necessary definitions and known results. From now on, let B be a nonemty set and let $(B,\cdot)$ be a semigroup, where $\cdot$ is a bynary operation.

A semigroup automaton is a triple $(S,(B,\cdot),f)$, where S is a set, $(B,\cdot)$ is a semogroup, and $f:S\times B\to S$ is a map satisfying

$$f(f(s,x),y)=f(s,x\cdot y), \tag{1}$$

for every $s\in S$, $x,y\in B$.

The set S is called the set of **states** of $(S,(B,\cdot),f)$ and f is called the **transition function** of $(S,(B,\cdot),f)$.

A nonempty set B with the (3,2)-operation $\{\ \}:B^3\to B^2$ is called **a (3,2)-semigroup** iff the following equality

$$\{\{xyz\}t\}=\{x\{yzt\}\} \tag{2}$$

is an identity for every $x,y,z,t\in B$. It is denoted with the pair $(B,\{\ \})$.

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Example 1: Let $B=\{a,b\}$. Then the (3,2)-semigroup $(B,\{\ \})$ is given by Table 1.

This example of (3,2)-semigroup is generated by an appropriate computer program.

TABLE 1

(3,2)- SEMIGROUP

{ }	
a a a	(b,a)
a a b	(a,a)
a b a	(a,a)
a b b	(b,a)
b a a	(a,a)
b a b	(b,a)
b b a	(b,a)
b b b	(a,a)

A (deterministic) (3,2)-automaton is a triple (S,B,f), where S and B are nonempty sets, and $f:S\times B^2\to S\times B$ is a map.

A (deterministic) (3,2)-semigroup automaton is a triple $(S,(B,\{\ \}),f)$ where S is a set, $(B,\{\ \})$ is a (3,2)-semogroup, and $f:S\times B^2\to S\times B$ is a map satisfying

$$f(f(s,x,y),z)=f(s,\{xyz\}), \tag{3}$$

for every $s\in S$, $x,y,z\in B$.

The set S is called the set of **states** of $(S,(B,\{\ \}),f)$ and f is called the **transition function** of $(S,(B,\{\ \}),f)$.

Example 2: Let $(B,\{\ \})$ be a (3,2)-semigroup given by Table 1 from Example 1 and $S=\{s_0,s_1,s_2\}$. A (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ is given by Table 2 and the graph in Fig. 1 .

TABLE 2

f	(a,a)	(a,b)	(b,a)	(b,b)
s_0	(s_1,b)	(s_2,b)	(s_2,b)	(s_1,b)
s_1	(s_1,b)	(s_0,a)	(s_2,b)	(s_1,b)
s_2	(s_2,b)	(s_0,b)	(s_1,b)	(s_2,b)

(3,2)-SEMIGROUP AUTOMATON

This example of (3,2)-semigroup automaton is generated by computer.

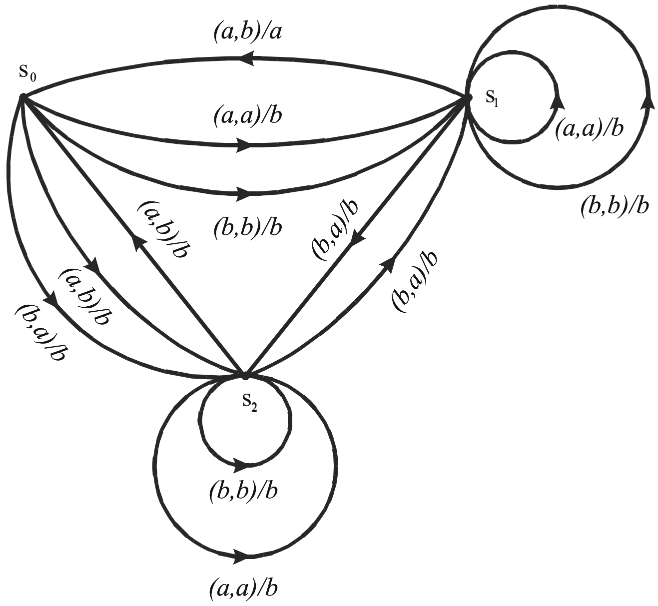

Fig. 1 (3,2)-semigroup automaton

Let $(Q.[\,])$ be a free (3,2)-semigroup with a basis B constructed in [1].

Any subset $L^{(3,2)}$ of the universal language $Q^* = \bigcup_{p\geq 1} Q^p$, where Q is a free (3,2)-semigroup with a basis B, is called **a (3,2)-language** on the alphabet B.

A (3,2)-language $L^{(3,2)} \subseteq Q^*$ is called **recognizable** if there exists:

(1) a (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$, where the set S is finite;

(2) an initial state $s_0 \in S$;

(3) a subset $T \subseteq S$; and

(4) a subset $C \subseteq B$ such that

$$L^{(3,2)} = \{w \in Q^* \mid \overline{\varphi}(s_0,(w,1),(w,2)) \in T \times C\}, \quad (4)$$

where $(S,(Q,[\ ]),\overline{\varphi})$ is the (3,2)-semigroup automaton constructed in [2] for the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$.

We also say that the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ **recognizes** $L^{(3,2)}$, or that $L^{(3,2)}$ **is recognized** by $(S,(B,\{\ \}),f)$.

Example 3: Let $(S,(B,\{\ \}),f)$ be a (3,2)-semigroup automaton given in Example 2. We construct the (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$ for the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$. A (3,2)-language $L^{(3,2)}$, which is recognized by the (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$, with initial state s_0 and terminal state (s_2,b) is

$L^{(3,2)} = \{w \in Q^* \mid w = w_1 w_2 \ldots w_q,\ q \geq 3,$

where $w_l = \begin{cases} (u_1^n, i),\ n \geq 3, u_\alpha \in Q \\ (a^* b^*)^* \end{cases}$, $l \in \{1,2,\ldots,q\}$, and:

a) If $i = 1$, then:

a1) $(u_1^n, 1) = a$, where

$\psi_{p-1}(u_1)\ldots\psi_{p-1}(u_n) = a^t b^j a^r b^h$ and $t + r = 2k$,

$t + j + r + h = n$, $t, j, r, h, k \in \{0,1,2,\ldots\}$, $k \geq 1$;

a2) $(u_1^n, 1) = b$, where

$\psi_{p-1}(u_1)\ldots\psi_{p-1}(u_n) = a^t b^j a^r b^h$ and $t + r = 2k + 1$,

$t + j + r + h = n$, $t, j, r, h, k \in \{0,1,2,\ldots\}$, $k \geq 1$;

b) If $i = 2$, then $(u_1^n, 2) = a$, where

$\psi_{p-1}(u_1)\ldots\psi_{p-1}(u_n) = (a^* b^*)^*$;

and finally $\psi_p(w_1)\ldots\psi_p(w_q) = b^*(ab^*)^{2k+1}$ $\}$.

III. NONDETERMINISTIC (3,2)-SEMIGROUP AUTOMATON

Now we introduce a powerful feature to (3,2)-semigroup automata. This feature is called nondeterminism, and is essentially the ability to change pairs of state and symbol in a way that is only partially determined by the current pair of state and symbol, and input pair symbols. That is, we shall now permit several possible "next pairs of state and symbol" for a given combination of current pair of state and symbol and input pair of symbols. The (3,2)-semigroup automaton, as it reads the input string, may choose at each step to go into any one of these legal next pairs of state and symbol; the choice is not determined by anything in our model, and is therefore said to be nondeterministic. On the other hand, the choice is not wholly unlimited either; only those next pairs of states and symbol are legal from a given pair of state and symbol with a given input pair of symbols can be chosen.

A nondeterministic (3,2)-semigroup automaton is a triple $(S,(B,\{\ \}),g)$ where S is a set, $(B,\{\ \})$ is a (3,2)-semogroup, $P(S \times B)$ is a subset of $S \times B$ and $g : S \times B^2 \to P(S \times B)$ is a map satisfying

$$g(g(s,x,y),z) = g(s,\{xyz\}), \quad (5)$$

for every $s \in S$, $x,y,z \in B$.

We denote that $g(X,y) = \bigcup_{(t,a)\in X} g(t,a,y)$ for $X \subseteq S \times B$.

In case the mapping g is such that for every $s \in S$, $(x,y) \in B^2$, $g(s,x,y)$ is either empty or a single element of $S \times B$, $(S,(B,\{\ \}),g)$ is called **a partial (3,2)-semigroup automaton**.

There is a simler process for transforming a partial (3,2)-semigroup automaton into a (deterministic) (3,2)-semigroup automaton by adding to S one extra state $z \in S$ which is a fixed point for $(S \cup \{z\},(B,\{\ \}),g')$ as follows:

$g'(s,x,y) = g(s,x,y)$ if $g(s,x,y) \in S \times B$;

$g'(s,x,y) = (z,a)$, for some $a \in B$, if

$$g(s,x,y) \notin S \times B;$$

$g'(z,x,y) = (z,a)$ for all $x, y \in B$.

Example 4: Let $B = \{a,b\}$, $S = \{s_0, s_1, s_2\}$. $(B,\{\ \})$ is a (3,2)-semigroup given by Table 3.

TABLE 3
(3,2)-SEMIGROUP

{ }	
a a a	a a
a a b	a b
a b a	b a
a b b	b b
b a a	b a
b a b	b b
b b a	b a
b b b	b b

$(S,(B,\{\ \}),g)$ is a nondeterministic (3,2)-semigroup automaton given by Table 4 and the graph in Fig. 2 .

TABLE 4
NONDETERMINISTIC (3,2)-SEMIGROUP AUTOMATON

g	(a,a)	(a,b)	(b,a)	(b,b)
s_0	(s_0,a), (s_1,a)	(s_1,b)	(s_2,b)	(s_0,b)
s_1	(s_0,a), (s_1,a)	(s_1,b)	(s_2,b)	(s_0,b)
s_2	(s_2,a)	(s_2,b)	(s_2,b)	(s_0,b)

A nondeterministic (3,2)-semigroup automaton $(S,(B,\{\ \}),g)$ is recognized the (3,2)-language

$$L^{(3,2)} = \{w \in B^* \mid w = \{aaa^*[aa^*(aa^*aa^*a)^*ba \cup \cup b(a \cup bb^*a)] \cup bbb^*a\}(a \cup bb^*a)^*\}.$$

Theorem: A (3,2)-language $L^{(3,2)} \subseteq Q^*$ is recognizable if there is nondeterministic (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$ with finite set of states S and subsets $I \subseteq S$, $T \subseteq S$ and $C \subseteq B$ such that

$$L^{(3,2)} = \{w \in Q^* \mid \overline{\varphi}(I,(w,1),(w,2)) \cap (T \times C) \neq \emptyset\}.$$

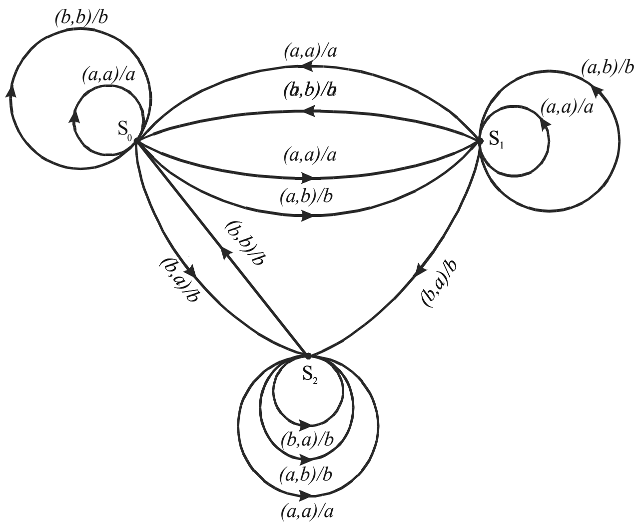

Fig. 2 (3,2)-semigroup automaton

Proof: Let $L^{(3,2)} \subseteq Q^*$ be (3,2)-language which is recognized by (deterministic) (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$ with finite set S, the initial state $s_0 \in S$ and subset $T \subseteq S$ and $C \subseteq B$ such that

$$L^{(3,2)} = \{w \in Q^* \mid \overline{\varphi}(s_0,(w,1),(w,2)) \in T \times C\}.$$

The deterministic (3,2)-semigroup automaton is a special case of nondeterministic (3,2)-semigroup automaton, so we have that there exists nondeterministic (3,2)-semigroup automaton with finite set of states S and subsets $I = \{s_0\} \subseteq S$, $T \subseteq S$ and $C \subseteq B$ such that

$$L^{(3,2)} = \{w \in Q^* \mid \overline{\varphi}(I,(w,1),(w,2)) \cap (T \times C) \neq \emptyset\}.$$

■

But the inverse is not true in each case.

Example 5: For the given nondeterministic (3,2)-semigroup automaton in Example 4, we construct (3,2)-automaton with introducing a new letter e in the alphabet B and a new states s_0', s_1' in the set of states S. The (3,2)-automaton $(S \cup \{s_0', s_1'\}, B \cup \{e\}, g')$ is given by Table 5 and Fig. 3 .

TABLE 5
(3,2)-AUTOMATON

f	(a,a)	(a,b)	(a,e)	(b,a)	(b,b)	(b,e)	(e,a)	(e,b)	(e,e)
s_0	(s_0,a)	(s_1,b)	(s_0',a)	(s_2,b)	(s_0',e)	(s_0',b)	(s_0',a)	(s_0',b)	(s_0,e)
s_0'	(s_1,a)	(s_0',e)	(s_0,a)	(s_0',e)	(s_0',e)	(s_0,b)	(s_0,a)	(s_0,b)	(s_0',e)
s_1	(s_0,a)	(s_1,b)	(s_1',a)	(s_2,b)	(s_0,b)	(s_1',b)	(s_1',a)	(s_1',b)	(s_1,e)
s_1'	(s_1,a)	(s_1',e)	(s_1,a)	(s_1',e)	(s_1',e)	(s_1,b)	(s_1,a)	(s_1,b)	(s_1',e)
s_2	(s_2,a)	(s_2,b)	(s_2,a)	(s_2,b)	(s_0,b)	(s_2,b)	(s_2,a)	(s_2,b)	(s_2,e)

It isn't (3,2)-semigroup automaton, because it isn't generated (3,2)-semigroup on the alphabet $B \cup \{e\}$. For example,

$$g(s_i, \{aaa\}) = g\begin{pmatrix} (s_0, a) & \\ (s_1, a) & \\ (s_0, a), & a \\ (s_1, a) & \\ (s_2, a) & \end{pmatrix} = \begin{pmatrix} (s_0, a) \\ (s_0, a) \\ (s_0, a) \\ (s_0, a) \\ (s_2, a) \end{pmatrix},$$

for $s_i \in S \cup \{s_0', s_1'\}$, but there aren't any pair (u, v), for $u, v \in B \cup \{e\}$, such that $\{aaa\} = (u, v)$.

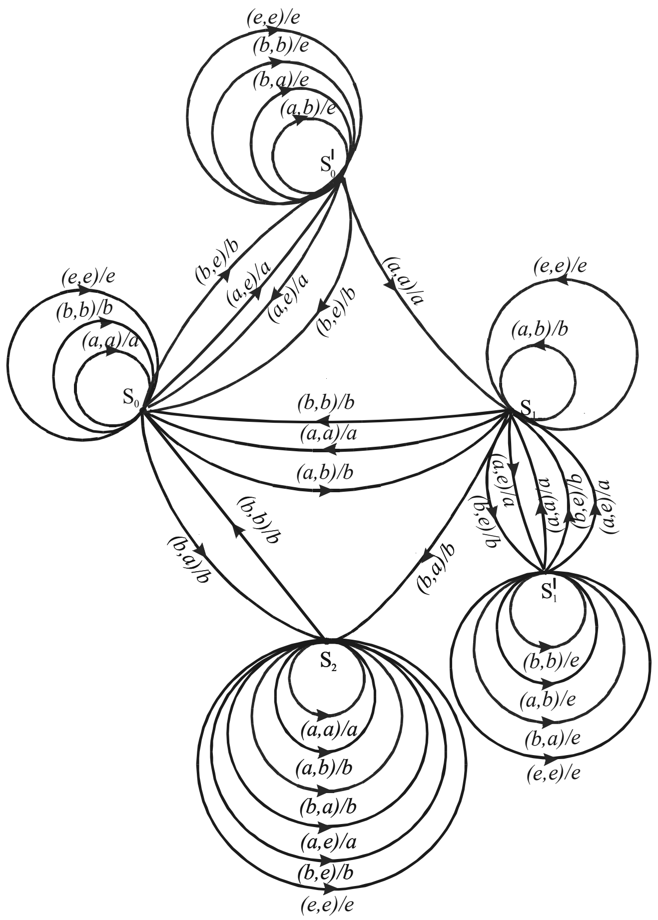

Fig. 3 (3,2)-automaton

IV. CONCLUSION

The results was given in this paper, are of the scientific interest, because they are a generalization of the semigroup automata in case (3,2). The Theorem was proved that every (3,2)-semigroup automaton can be transform into nondeterministic (3,2)-semigroup automaton with the appropriate method. But, the Example 5 was showed that the inverse is not true in each case.

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