Statistical Estimation Method of Electronic Linear Stationary System NON-Stability

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Abstract-**The paper presents a method of estimating the transient coefficient non-stability under the impact of complex factors with limited value variation. An active experiment has been used during the calculations and measurements. The mathematical models that have been developed by means of the obtained non-stability values can be used in synthesis and study on linear stationary systems in telecommunications and other fields. The method gives a possibility to determine and use the statistical extremum.**

The non-stability of linear system parameters is of great importance for its functioning. It is usual to give an admissible value that should not be exceeded. Otherwise the system goes out of its equilibrium state or stops performing the functions assigned, i.e. a failure occurs. The relevant operational system area is formed by the given non-stability of a determined indicators multitude which violence is connected again with a failure.

There are well-known methods of determining the non-stability related to certain system parameters and to the totality of them [1, 5]. The approaches are mainly analytical and sometimes are not very convenient with synthesis, simulation and experimental study.

The paper presents a method of estimating the linear system transient coefficient non-stability determined by the impact of a combination of factors which values vary in given boundaries. For the purpose an active experiment of the system simulation or of its experimental study has been used.

The linear continuous stationary system transient coefficient is determined by the ratio of reaction $\dot{S}_{out}(\omega)$ and impact $\dot{S}_{in}(\omega)$ i.e.

$$\dot{T}(\omega) = \frac{\dot{S}_{out}(\omega)}{\dot{S}_{in}(\omega)} . \qquad (1)$$

It is known that the modulus and argument of (1) determine the relevant amplitude and frequency feature as well as the relevant phase and frequency feature.

An analogous dependency can be obtained for a linear discrete stationary system where the values of its features are determined for the discrete values of time and frequency.

The transient coefficient depends on the system elements and the outer impacts:

$$T(\omega) = f_1(x_1, x_2, ..., x_n) . \qquad (2)$$

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For the linear stationary system dependency (2) is linear. The variations of the transient coefficient are relatively

$$\Delta T = f_2(\Delta x_1, \Delta x_2, ..., \Delta x_n) . \qquad (3)$$

The relative non-stability expressed also by those factors and their variations

$$\delta = \frac{\Delta T}{T} , \qquad (4)$$

has been used with the examination on the systems.

A mathematical model of non-stability can be obtained by active experiment [2, 6]. The model analysis is useful for the system study. Besides that he is adequate to determine the statistical extremum of the relative non-stability.

A linear model can be used with small variations of the variables $x_i \quad (i = \overline{1, n})$ .Thus the problem is simplified.

The number of variables can be reduced if the Pareto's principle is applied. According to him, the influence of the variables decreases in hyperbolic dependency. This problem is solved by the active experiment.

To develop a linear model it is necessary to standardize the variables according to the dependency:

$$z_i = \frac{x_i - x_{i\,OCH}}{\Delta x_i} \quad (i = \overline{1, n}) , \qquad (5)$$

where z_i is the standardized variable;

$x_{i\,OCH}$ – the initial values of the variables (parameters of the building elements, voltages, etc.);

Δx_i – changes of variables that is expedient to be assumed in compliance with the nominal values of standard resistors, condensers, etc.

If calculations or measurements of T with $x_i' = x_{i\,OCH} + \Delta x_i$ and $x_i'' = x_{i\,OCH} - \Delta x_i$ are carried out for z_i +1 and –1 are obtained respectively. This operation makes the model development and analysis easier. It is

$$T = b_0 + b_1 z_1 + b_2 z_2 + ... + b_n z_n . \qquad (6)$$

Coefficients $b_i \quad (i = \overline{0, n})$ characterize the influence of each variable. The value of b_0 corresponds to value of T_0 in the initial state. If the sign in front of b_i is positive, it shows the increase of *T* with the increase of z_i and vise versa, it shows the decrease of *T* if it is negative.

where the regression coefficients are determined by the formula

Table 1

u	z_0	z_1	z_2	z_3	T	$\Delta T = \lvert T - T_0 \rvert$	$\delta = \Delta T / T_0$
0		0	0	0	T_0		
1	1	-1	-1	-1	T_1	ΔT_1	δ_1
2	1	-1	-1	1	T_2	ΔT_2	δ_2
3	1	-1	1	-1	T_3	ΔT_3	δ_3
4	1	-1	1	1	T_4	ΔT_4	δ_4
5	1	1	-1	-1	T_5	ΔT_5	δ_5
6	1	1	-1	1	T_6	ΔT_6	δ_6
7	1	1	1	-1	T_7	ΔT_7	δ_7
8	1	1	1	1	T_8	ΔT_8	δ_8

The value of b_i shows the influence rate of z_i.The numerical comparison of all coefficients admits то neglect some of the variables and simplify the model. To precise this operation the criterion of Student has been used [2, 6].

When it is necessary to present (6) by real variables, the meanings of (5) are replaced in this expression and relevant processing is done in order to obtain dependency (2).

T_0, ΔT and $\delta = \Delta T / T_0$ are determined using the data obtained calculations or measurements. Thus, for example, a possibility has been created to solve a problem that often appears in practice: determining the depth of the negative feedback *F* with a given relative non-stability of transient coefficient δ', i.e.

$$F = 1 + \beta T_0 = \frac{\delta}{\delta'}, \tag{7}$$

where β is the feedback coefficient determined by the ratio between the signal at the system input S'_{in} and the signal at its output S'_{out} $(\beta = S'_{in} / S'_{out})$.

The experimental values of $\delta = \Delta T / T_0$ can be determined by the model and that is connected with obtaining $\delta = \delta_{min}$ and the selection of building materials and voltages with relevant values.

The statistical extremum is calculated using the data of active experiment. They correspond to the reports on the observation of objects [3, 4]. The time spent for observation is reduced by the active experiment.

Table 1 is an exemplary matrix of the experiment with variables z_1, z_2 and z_3.

The exemplary linear model of δ is in the kind of

$$\delta = b_0 + b_1 z_1 + b_2 z_2 + b_3 z_3, \tag{8}$$

$$b_i = \frac{\sum_{n=1}^{8} x_{in} \delta_n}{8} \quad (i = 0, 1, 2, 3). \tag{9}$$

The models of *T* and ΔT can be developed in analogous way.

The regression coefficients are significant if the condition

$$b_i \geq t \frac{S_\delta}{\sqrt{8}} \tag{10}$$

has been kept, where *t* is the table value of the criterion of Student with the level of significance chosen and the relevant rates of freedom; S_δ the disperse of reproduction

$$S_\delta = \frac{\sum_{k}^{m} (\delta_k - \bar{\delta})^2}{m - 1}. \tag{11}$$

The disperse of reproduction is determined by *m* parallel computations or measurements that can be carried out with a zero level of variables at the beginning.

The number of experiments (computations or measurements) in the example shown is $N = 2^3 = 8$. They are $N = 2^n$ with *n* variables but can be canceled by a fraction retort or by a saturated project [2, 6].

To determine the regression coefficients it is necessary to do *n*+1 experiments and one more to estimate the model adequacy. A model of the kind of

$$\delta = b_0 + b_1 z_1 + b_2 z_2 + b_3 z_3 + b_{12} z_1 z_2 + b_{13} z_1 z_3 + b_{23} z_2 z_3 \tag{12}$$

can be developed for the particular case.

The interactions of variables are estimated by coefficients b_{12}, b_{13}, b_{23}.

The statistical extremum is very suitable to estimate risk situations. The system non-stability is directly connected with a certain risk of failure. The values of $\delta_u \quad (u = \overline{0, N})$ are assumed to be terms of the series. For particular case they are in the last column of Table 1. To do computations the following formulas are used:

a) with seeking maximum

$$P(y) = \exp(-e^{-y}); \quad y = \frac{\delta - \mu}{\sigma}; \quad \mu = \overline{\delta} - 0{,}557\sigma; \tag{13}$$

b) with seeking minimum

$$P(y) = 1 - \exp(-e^{-y}); \quad y = \frac{\delta - \mu}{\sigma}; \quad \mu = \overline{\delta} + 0{,}557\sigma. \tag{14}$$

The probability of maximum and minimum appearance is determined by *P(y)*. The dependencies

$$\overline{\delta} = \frac{\sum_{u=1}^{N} \delta_u}{N}; \quad \sigma = 1{,}283\sqrt{D}; \quad D = \frac{\sum_{u=1}^{N} (\delta_u - \overline{\delta})^2}{N-1}, \tag{15}$$

are used for $\overline{\delta}$ and σ where N is the number of the statistic series terms; δ – the non-stability extreme value.

Two problems can be formulated:

1. δ is given and the probability not exceeding this value is sought;

2. A value of δ that will not be exceeded is determined with a given probability *P(y)*.

The method proposed can find application in development of electronic systems with high stability of parameters.

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