Power System Elements Modeling in Sequence Domain

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Abstract – **In this paper the new concept for power system elements modeling in sequence domain is presented. The proposed models are decoupled either case for balanced or unbalanced elements. Mathematically, 6x6 sparse node-admittance matrices describe the generators, transformers and lines. The load model is given as injected sequence complex currents (or powers) in adequate nodes of sequence circuits.**

Keywords – **Elements modeling, Sequence domain, Decoupling, Asymmetrical load-flow.**

I. Introduction

Both symmetrical load-flow and faulted power system studies are performed in sequence domain. This domain is more efficient and comfortable than of the phase domain. The efficiency and comfort of the sequence domain are the result of diagonal forms of matrix representatives of balanced power systems elements (mutual couplings between phases are eliminated). These elements very frequently appear in typical power systems (practically all generators and transformers, transposed lines, symmetrical loads, etc.). In this case, the circuits representing positive-, negative- and zero-sequence models of the entire power system in the most number of cases can be represented separately, without influences to each other.

Symmetrical states are good approximations of usual states of three-phase power systems. But actually, because of the presence of long unbalanced (untransposed) lines, asymmetrical or single-phase loads (as induction furnaces and traction motors etc.), asymmetrical states in power systems are occurred.

For more precise analysis of three-phase power system asymmetrical states, the asymmetrical load-flow (ALF) analysis is required. Usually, the solution of ALF problem is performed using methods in phase domain. In this domain, (because of mutual couplings between phases) 6x6 node-admittance matrices which describe the generators, transformers and lines are not sparse. This fact implies increasing of both memory for problem storage and CPU time for problem solution in the phase domain against solutions in the sequence domain.

It seems that the main reasons for avoiding the sequence domain in the ALF methods are the problems of elements modeling because of: 1 – phase shifts of three-phase transformers (ideal transformers with complex turns ratios in their sequence circuits); 2 – the coupling existing between sequence circuits for unbalanced lines model and 3 – asymmetrical phase loads, which cannot be specified in the sequence domain.

First solutions of the ALF problem in sequence domain are proposed in [1]. Although the problems of unbalanced long lines and load modeling in sequence domain are solved, the problem of ideal transformers with complex turn ratio modeling still existed. The transformer model in sequence domain is obtained by transformation of its model in the phase domain. Thus, the advancements of direct modeling in sequence domain and application of 6x6 sparse matrices are lost. Applying the elements models presented in this paper, several new efficient methods for ALF analysis in sequence domain was established in [2,3], where the domination of the methods in sequence domain against the methods in phase domain is confirmed in any way.

II. Generator Model

The generator is balanced element of the power system. In the sequence domain it can be presented with three decoupled sequence circuits. Scaled sequence circuits of a synchronous generator are presented in Fig.1a-c (the superscript of positive-sequence parameters is d, for negative-sequence is i and for zero-sequence is o; the internal and the external bus are signed by 1 and 2, respectively).

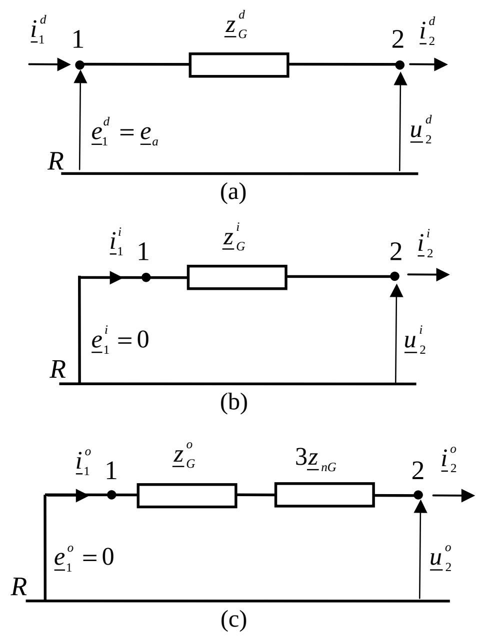

Fig. 1. Synchronous generator scaled positive- (a), negative- (b) and zero-sequence circuit (c).

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The scaled sequence impedances of the synchronous generator are denoted by $\underline{z}_G^d$, $\underline{z}_G^i$ and $\underline{z}_G^o$; $\underline{z}_{nG}$ represents the generator grounding impedance; the phase *a* generator open-circuit voltage is denoted by $\underline{e}_a$; it is equal to the positive-sequence internal bus voltage. Usually, instead of sequence impedances, the sequence admittances defined with Eqs. (1) are used.

$$\underline{y}_G^d = 1/\underline{z}_G^d \;; \quad \underline{y}_G^i = 1/\underline{z}_G^i \;; \quad \underline{y}_G^o = 1/\left(\underline{z}_G^o + 3\underline{z}_{nG}\right) \qquad (1)$$

In order to get the sequence domain mathematical model for the synchronous generator, it is necessary to establish the nodal voltage equations for the sequence circuits. Taking into account the marks from the Fig. 1, these equations are given by Eq. (2) in the matrix form:

$$\begin{bmatrix} \underline{i}_1^d \\ \underline{i}_1^i \\ \underline{i}_1^o \\ \hline -\underline{i}_2^d \\ -\underline{i}_2^i \\ -\underline{i}_2^o \end{bmatrix} = \left[\begin{array}{ccc|ccc} \underline{y}_G^d & 0 & 0 & -\underline{y}_G^d & 0 & 0 \\ 0 & \underline{y}_G^i & 0 & 0 & -\underline{y}_G^i & 0 \\ 0 & 0 & \underline{y}_G^o & 0 & 0 & -\underline{y}_G^o \\ \hline -\underline{y}_G^d & 0 & 0 & \underline{y}_G^d & 0 & 0 \\ 0 & -\underline{y}_G^i & 0 & 0 & \underline{y}_G^i & 0 \\ 0 & 0 & -\underline{y}_G^o & 0 & 0 & \underline{y}_G^o \end{array}\right] \begin{bmatrix} \underline{e}_1^d \\ \underline{e}_1^i \\ \underline{e}_1^o \\ \hline \underline{u}_2^d \\ \underline{u}_2^i \\ \underline{u}_2^o \end{bmatrix} \qquad (2)$$

It is obvious that the node-admittance matrix representing the synchronous generator in the sequence domain is sparse matrix. This is not the same case in the phase domain. If 6x6 generator node-admittance matrices in phase and sequence domain are compared, it would be concluded that in sequence domain this matrix consists 33.3% non-zero elements, but in phase domain all elements have non-zero values [3].

III. TRANSFORMER MODEL

The three-phase transformer can be treated as a balanced element of the power system. In accordance with the winding connections, the transformer introduces phase shifting between the voltages (currents) on the each transformer side.

When symmetrical states are considered, the phase shifts of three-phase transformers can be ignored because of complex powers, currents and voltages of all phases are uniformly transferred from one to the other transformer side, independently of the transformer state. In contrast, when asymmetrical states are considered, these transfers are neither uniform nor independent of the transformer state.

Although the model of three-phase transformer with complex turn ratio can be presented with the three decoupled sequence circuits, the problem of complex turn ratio still exists. As an example, Y-Δ-1 connected transformer sequence circuits are depicted in Fig. 2a-c. $\underline{Z}_T$ and $\underline{Z}_{nT}$ denote the transformer positive-sequence and grounding impedances, respectively. $\underline{U}$ and $\underline{I}$, with corresponding superscripts, denote complex voltages and currents. V_{n1} and V_{n2} are transformer nominal voltages. The transformer positive-sequence phase shift amounts +π/6 and the negative-sequence is –π/6; the transformer zero-sequence phase shift does not have to be introduced.

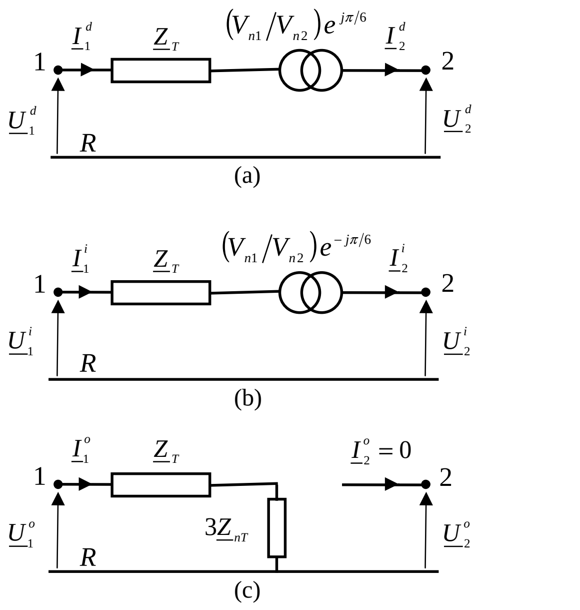

Fig. 2. Y-Δ-1 connected three-phase transformer positive- (a), negative- (b) and zero-sequence circuit (c) in absolute value domain.

Application of the standard PU system for transferring the values from the absolute to the relative value domain doesn't solve the problem of phase shifting.

Ideal transformers with complex turns ratios disturb the symmetry and the simplicity of deriving power system node-admittance matrices in the sequence domain.

Difficulties that three-phase transformer phase shifts introduce into sequence domain models are definitely eliminated in [4] by applying "New Scaling Concept". It was first used in [5] for evaluating a very successful solution of (unbalanced) power system problems with complex faults in the sequence domain. The result of scaling the absolute values with this concept is scaled sequence circuits of a Y-Δ-1 transformer, presented in Fig. 3a-c.

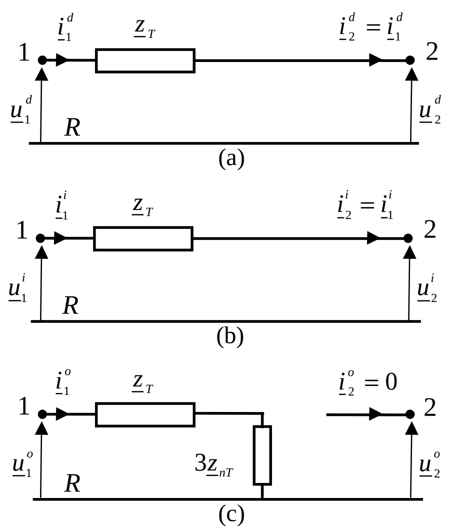

Fig. 3. Y-Δ-1 connected three-phase transformer positive- (a), negative- (b) and zero-sequence circuit (c) in relative value domain.

This figure shows that the complex turns ratios (modules and phase shifts) of the considered Y-Δ-1 three-phase transformer are eliminated in the sequence domain. Now, the nodal voltage equations for the transformer simplified sequence circuits can be written in matrix form with Eq. (3):

$$\begin{bmatrix} \underline{i}_1^d \\ \underline{i}_1^i \\ \underline{i}_1^o \\ -\underline{i}_2^d \\ -\underline{i}_2^i \\ -\underline{i}_2^o \end{bmatrix} = \begin{bmatrix} \underline{y}_T & 0 & 0 & -\underline{y}_T & 0 & 0 \\ 0 & \underline{y}_T & 0 & 0 & -\underline{y}_T & 0 \\ 0 & 0 & \underline{y}_T^o & 0 & 0 & 0 \\ -\underline{y}_T & 0 & 0 & \underline{y}_T & 0 & 0 \\ 0 & -\underline{y}_T & 0 & 0 & \underline{y}_T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{u}_1^d \\ \underline{u}_1^i \\ \underline{u}_1^o \\ \underline{u}_2^d \\ \underline{u}_2^i \\ \underline{u}_2^o \end{bmatrix} \quad (3)$$

In sequence domain the Y-Δ-1 transformer 6x6 node-admittance matrix has only 25% non-zero elements, but in phase domain this matrix has 66.7% non-zero elements [3].

The node-admittance matrix depends of the transformer windings connection and value of the grounding impedances. Anyway, it is obvious that 6x6 node-admittance matrices representing the three-phase transformers with complex turn ratios in the sequence domain will be sparse matrices.

IV. LINE MODEL

Transmission overhead lines can be treated as balanced (transposed lines) or unbalanced (untransposed high-voltage long lines) power system elements. If the line is balanced element, the line model in the sequence domain can be presented with three lumped-π decoupled sequence circuits. Each circuit consists series admittance between line ends and two equal shunt admittances at the line ends. Consequently, 6x6 node-admittance matrix for the line model will be sparse.

When the unbalanced lines are considered in sequence domain, there are couplings among positive-, negative- and zero-sequence and the line model cannot be presented with lumped-π decoupled sequence circuits. In this case, 6x6 node-admittance matrix is full with non-zero elements, just like the 6x6 node-admittance matrix in the phase domain. Thus, the power system model in sequence domain cannot be presented with three linear decoupled sequence circuits. The points of mutually coupling among positive-, negative- and zero-sequence power system circuits are just these unbalanced lines. Applying compound matrix notation, the unbalanced line in sequence domain is presented in Fig. 4.

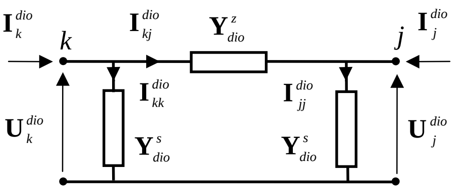

Fig. 4. Unbalanced line compound lumped-π model in sequence domain.

Inductive and capacitive mutual couplings among positive-, negative- and zero-sequence are expressed with non-zero off-diagonal elements in matrices:

$$\mathbf{Y}_{dio}^z = \begin{bmatrix} \underline{Y}_{dd}^z & \underline{Y}_{di}^z & \underline{Y}_{do}^z \\ \underline{Y}_{id}^z & \underline{Y}_{ii}^z & \underline{Y}_{io}^z \\ \underline{Y}_{od}^z & \underline{Y}_{oi}^z & \underline{Y}_{oo}^z \end{bmatrix} \text{ and } \mathbf{Y}_{dio}^s = \begin{bmatrix} \underline{Y}_{dd}^s & \underline{Y}_{di}^s & \underline{Y}_{do}^s \\ \underline{Y}_{id}^s & \underline{Y}_{ii}^s & \underline{Y}_{io}^s \\ \underline{Y}_{od}^s & \underline{Y}_{oi}^s & \underline{Y}_{oo}^s \end{bmatrix}.$$

Instead of mutually admittances, the couplings can be expressed by compensation current sources. Thus, the unbalanced line model can be presented with three decoupled sequence circuits as it is depicted in Fig. 5a-c. The mutual couplings are replaced by corresponding controlled sources – current sources.

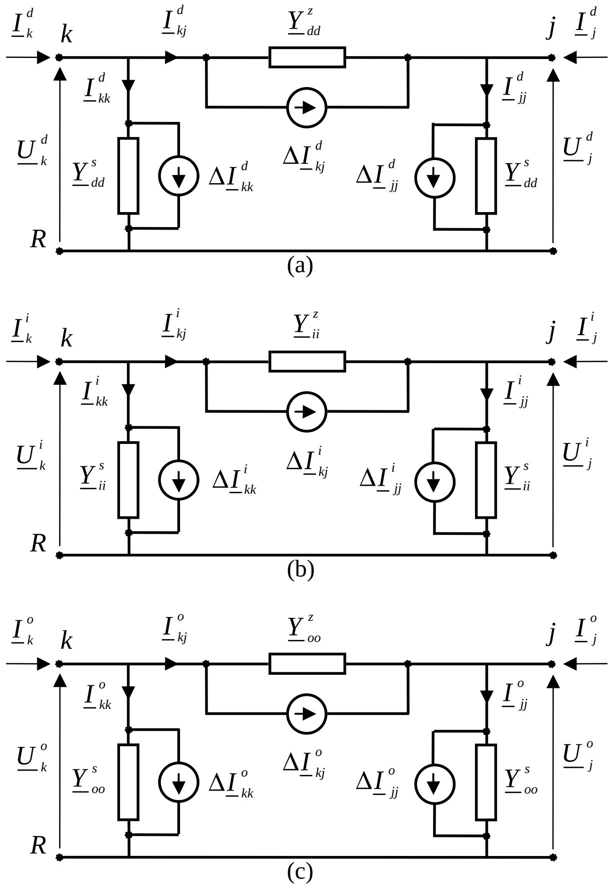

Fig. 5. Unbalanced line decoupled positive- (a), negative- (b) and zero-sequence circuit (c) in absolute value domain.

The current controlled sources in series and shunt branches of each sequence lumped-π circuit include the coupling influences from the other sequences. If the notation (*m*, *l*, *n*)=(*d*, *i*, *o*) where *m*≠*l*≠*n* is used, the self-admittance and the current source currents in series branch of any sequence from the Fig. 5 can be calculated by Eqs. (4) and (5), respectively.

$$\underline{Y}_{mm}^z \left(\underline{U}_k^m - \underline{U}_j^m\right) = \underline{I}_{kj}^m - \Delta\underline{I}_{kj}^m, \quad (4)$$

$$\Delta\underline{I}_{kj}^m = \underline{Y}_{ml}^z \left(\underline{U}_k^l - \underline{U}_j^l\right) + \underline{Y}_{mn}^z \left(\underline{U}_k^n - \underline{U}_j^n\right), \quad (5)$$

Similarly, the self-admittance and the current source currents in shunt branches of any sequence can be calculated by Eqs. (6) and (7) for branch in node *k*, and Eqs. (8) and (9) for branch in node *j*.

$$\underline{Y}^{s}_{mm}\underline{U}^{m}_{k} = \underline{I}^{m}_{kk} - \Delta\underline{I}^{m}_{kk}, \tag{6}$$

$$\Delta\underline{I}^{m}_{kk} = \underline{Y}^{s}_{ml}\underline{U}^{l}_{k} + \underline{Y}^{s}_{mn}\underline{U}^{n}_{k}, \tag{7}$$

$$\underline{Y}^{s}_{mm}\underline{U}^{m}_{j} = \underline{I}^{m}_{jj} - \Delta\underline{I}^{m}_{jj}, \tag{8}$$

$$\Delta\underline{I}^{m}_{jj} = \underline{Y}^{s}_{ml}\underline{U}^{l}_{j} + \underline{Y}^{s}_{mn}\underline{U}^{n}_{j}, \tag{9}$$

Taking into account the currents directions in the sequence circuits depicted in Fig. 5a-c, the compensation currents expressed by Eq. (10) for node k and Eq. (11) for node j, can be defined.

$$\Delta\underline{I}^{m}_{k} = -\Delta\underline{I}^{m}_{kk} - \Delta\underline{I}^{m}_{kj}, \tag{10}$$

$$\Delta\underline{I}^{m}_{j} = -\Delta\underline{I}^{m}_{jj} + \Delta\underline{I}^{m}_{kj} \tag{11}$$

Now, the injected currents in the ends of any sequence lumped-π circuit can be corrected by above defined compensation currents. These corrections enable the omition of the current sources from the sequence circuits in Fig. 5a-c and obtaining the final decoupled, compensated, scaled, unbalanced line model in sequence domain depicted in Fig. 6.

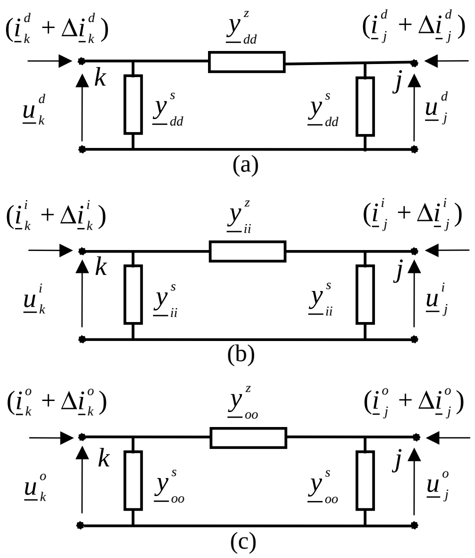

Fig. 6. Unbalanced line decoupled, compensated, scaled positive- (a), negative- (b) and zero-sequence (c) lumped-π circuits.

The 6x6 node-admittance matrix representing the unbalanced line in sequence domain has the following form:

$$\left[\begin{array}{ccc|ccc}
\underline{y}^{z}_{dd}+\underline{y}^{s}_{dd} & 0 & 0 & -\underline{y}^{z}_{dd} & 0 & 0 \\
0 & \underline{y}^{z}_{ii}+\underline{y}^{s}_{ii} & 0 & 0 & -\underline{y}^{z}_{ii} & 0 \\
0 & 0 & \underline{y}^{z}_{oo}+\underline{y}^{s}_{oo} & 0 & 0 & -\underline{y}^{z}_{oo} \\
\hline
-\underline{y}^{z}_{dd} & 0 & 0 & \underline{y}^{z}_{dd}+\underline{y}^{s}_{dd} & 0 & 0 \\
0 & -\underline{y}^{z}_{ii} & 0 & 0 & \underline{y}^{z}_{ii}+\underline{y}^{s}_{ii} & 0 \\
0 & 0 & -\underline{y}^{z}_{oo} & 0 & 0 & \underline{y}^{z}_{oo}+\underline{y}^{s}_{oo}
\end{array}\right]$$

This matrix is sparse and has the same form as the 6x6 node-admittance matrix representing balanced line in sequence domain.

V. LOAD MODEL

Usually, load active and reactive powers for each phase (a, b and c) are specified. For example in the power system bus k, these powers are denoted as:

$$P^{m}_{k} = P^{m}_{kspec};\ Q^{m}_{k} = Q^{m}_{kspec};\quad m=a,b,c \tag{12}$$

The individual complex currents and powers of the loads in positive-, negative and zero-sequence can be derived if the phase complex voltages $\mathbf{U}^{abc}_{k}$ or sequence complex voltages $\mathbf{U}^{dio}_{k}$ are on disposal. The phases injected complex currents in the bus k, are:

$$\underline{I}^{m}_{k} = \frac{-(P^{m}_{k} + jQ^{m}_{k})^{*}}{\left(\underline{U}^{m}_{k}\right)^{*}};\quad m=a,b,c \tag{13}$$

With known complex currents in phase domain and inverse transformation matrix [2], injected complex currents in node k in any sequence circuit are expressed as:

$$\mathbf{I}^{dio}_{k} = \mathbf{T}^{-1}_{s}\mathbf{I}^{abc}_{k} \tag{14}$$

Matrix Eq. (14) represents the load model in sequence domain. Also, using the sequence complex voltages and currents, the sequence complex powers can be calculated. Thus, the load model can be presented through injected sequence complex powers.

VI. CONCLUSION

In this paper the procedure of power system balanced or unbalanced elements modeling in sequence domain is presented. Applying the "New scaling concept", the phase shifts introduced by the ideal transformers with complex turn ratio are eliminated from the sequence circuits. With current compensation procedure the unbalanced line is modeled with decoupled sequence models. Thus, generator, transformer and line decoupled models are represented with 6x6 node-admittance sparse matrices. Injected sequence complex currents or powers in the busbar where the load is connected represent the load model. These elements models enable unbalanced power system modeling with three sequence, decoupled circuits.

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