A Simplified Linearized Dynamic Model For Voltage Collapse Assessment In Power Systems

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Abstract - **In this paper, a simplified linearized dynamic model for fast assessment of voltage collapse is developed. The simplification was made under assumption that the voltages at generator nodes are constant, which means that only power changes at load nodes are considered in analysis. Dimensions of the state matrix, which eigenvalues are used for voltage collapse assessment, decrease under this assumption. Appropriate transformations of linearized state matrix prove that for this model knowing of values of the dynamical load change time constants are not required.**

Keywords – **voltage collapse, assessment, load, state matrix.**

I. INTRODUCTION

Analysis of faults caused by voltage collapse, and its consequences, pointed out this phenomenon is very complex and influenced by many factors. Because of this complexity, for a long time this phenomenon occupies interest of researchers, what verify a number of published papers. Different approaches are used in voltage collapse researches, depending on: emphasised factors, used models of components, and introduced simplifications. In general, all approaches for analysis of voltage collapse and voltage (in)stability can be classified into two basic groups: static and dynamic.

Linearized models are often used in both static [1,2,5,6] and dynamic [1,2,7-10] approaches. Eigenvalues of related linearized matrix are then used for voltage collapse presence indication.

In this paper, a simplified linearized dynamic model for voltage collapse assessment is formed. This model is obtained under assumption that voltages at generator nodes are constant. That means that only power changes at load nodes can be considered in analysis. Starting from this model, the voltage collapse is assessed by the eigenvalues of linearized state matrix. Dimensions of this matrix are equal to the total number of load nodes.

II. DYNAMIC OF POWER CHANGE AT LOAD NODES

The system under consideration has *m* generator and *n* load nodes. Dynamic of load changes at the *i*-th node can be expressed by following two differential equations [8,10]:

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$$\frac{dP_{Li}}{dt} = -\frac{1}{T_i}(P_{Li} - f_{Pi}(V_i)) \tag{1}$$

$$\frac{dQ_{Li}}{dt} = -\frac{1}{T_i}(Q_{Li} - f_{Qi}(V_i)) \tag{2}$$

where P_{Li} is active load at *i*-th node, Q_{Li} reactive load at *i*-th node, T_i dynamical load change time constant at *i*-th node, $f_{Pi}(V_i)$ dependence of the active load at *i*-th node as a function of the voltage V_i, and $f_{Qi}(V_i)$ dependence of the reactive load at *i*-th node as a function of the voltage V_i.

Time constant T_i depends on the load structure. Major factor that influences on this value is time constant of asynchronous machine, which represents loads in proposed model. Value of T_i also depends on time constant of tap changing transformer regulator, if it is presented at load node. Determination of the time constant T_i is very complex problem for each particular case. Proposed approach does not require knowing of time constants values T_i, which can be considered as an advantage.

Functional relations of f_{Pi} and f_{Qi} are static voltage characteristics of load at *i*-th node. In previous papers [1-4], different methods for modeling of static voltage characteristics are presented. In this paper, following functional relations are used:

$$f_{Pi}(V_i) = P^o_{Li}\left(\frac{V_i}{V_o}\right)^{k_{pvi}} \tag{3}$$

$$f_{Qi}(V_i) = Q^o_{Li}\left(\frac{V_i}{V_o}\right)^{k_{qvi}} \tag{4}$$

where (for *i*-th node) are k_{pvi}, k_{qvi} voltage selfregulation coefficients of the active and reactive load, P^o_{Li} active and Q^o_{Li} reactive loads that correspond to voltage V_0.

III. LINEARIZED DYNAMIC MODEL

Analysis of the voltage collapse appearance, as usually, starts from known initial conditions, i.e. from initial values of the voltage phasors at all nodes. Moreover, constant magnitudes of voltages at the generator nodes are assumed, while magnitudes of the voltages at load nodes are treated as corresponding functions of the active and reactive loads. Small changes in voltages ΔV, then can be expressed as

$$\Delta V = \left.\frac{\partial V}{\partial P_L}\right|_0 \Delta P_L + \left.\frac{\partial V}{\partial Q_L}\right|_0 \Delta Q_L \tag{5}$$

where:

$$\Delta V = [\Delta V_1\ \Delta V_2 \cdots \Delta V_n]^T$$
$$\Delta P = [\Delta P_{L1}\ \Delta P_{L2} \cdots \Delta P_{Ln}]^T$$
$$\Delta Q = [\Delta Q_{L1}\ \Delta Q_{L2} \cdots \Delta Q_{Ln}]^T$$

$$\left.\frac{\partial V}{\partial P_L}\right|_0 = \begin{bmatrix} \frac{\partial V_1}{\partial P_{L1}} & \cdots & \frac{\partial V_1}{\partial P_{Ln}} \\ \vdots & & \vdots \\ \frac{\partial V_n}{\partial P_{L1}} & \cdots & \frac{\partial V_n}{\partial P_{Ln}} \end{bmatrix}_0 , \quad \left.\frac{\partial V}{\partial Q_L}\right|_0 = \begin{bmatrix} \frac{\partial V_1}{\partial Q_{L1}} & \cdots & \frac{\partial V_1}{\partial Q_{Ln}} \\ \vdots & & \vdots \\ \frac{\partial V_n}{\partial Q_{L1}} & \cdots & \frac{\partial V_n}{\partial Q_{Ln}} \end{bmatrix}_0$$

Subscripts "0" in these equations denote that partial derivatives are calculated for steady state before changes appear.

Linearizing the functions $f_{Pi}(V_i)$ and $f_{Qi}(V_i)$ around the analysed initial state and respecting Eq. (5), Eqs. (3) and (4) became:

$$f_{Pi}(V_i) = P_{Li}(0) + \left(\frac{\partial f_{Pi}}{\partial V_i}\frac{\partial V_i}{\partial P_L}\right)_0 \Delta P_L + \left(\frac{\partial f_{Pi}}{\partial V_i}\frac{\partial V_i}{\partial Q_L}\right)_0 \Delta Q_L \tag{6}$$

$$f_{Qi}(V_i) = Q_{Li}(0) + \left(\frac{\partial f_{Qi}}{\partial V_i}\frac{\partial V_i}{\partial P_L}\right)_0 \Delta P_L + \left(\frac{\partial f_{Qi}}{\partial V_i}\frac{\partial V_i}{\partial Q_L}\right)_0 \Delta Q_L \tag{7}$$

where are $P_{Li}(0)$ initial steady-state active load, $Q_{Li}(0)$ initial steady-state reactive load at the *i*-th node, and:

$$\left.\frac{\partial V_i}{\partial P_L}\right|_0 = \left[\frac{\partial V_i}{\partial P_{L1}}\ \frac{\partial V_i}{\partial P_{L2}} \cdots \frac{\partial V_i}{\partial P_{Ln}}\right]_0$$
$$\left.\frac{\partial V_i}{\partial Q_L}\right|_0 = \left[\frac{\partial V_i}{\partial Q_{L1}}\ \frac{\partial V_i}{\partial Q_{L2}} \cdots \frac{\partial V_i}{\partial Q_{Ln}}\right]_0$$

Power increments at *i*-th node can be expressed by following:

$$\Delta P_{Li} = P_{Li} - P_{Li}(0) \tag{8}$$
$$\Delta Q_{Li} = Q_{Li} - Q_{Li}(0) \tag{9}$$

System of linearized differential equations is obtained if Eqs. $(6)-(9)$ are sequentially substituted in Eqs. (1) and (2). For the case of *n* load nodes network, following system of linearized differential equation can be written in matrix form:

$$\frac{d}{dt}\begin{bmatrix}\Delta P_L \\ \Delta Q_L\end{bmatrix} = \begin{bmatrix} -T^{-1}\left(I - \left(\frac{\partial f_P}{\partial V}\frac{\partial V}{\partial P_L}\right)_0\right) & T^{-1}\left(\frac{\partial f_P}{\partial V}\frac{\partial V}{\partial Q_L}\right)_0 \\ T^{-1}\left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial P_L}\right)_0 & -T^{-1}\left(I - \left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial Q_L}\right)_0\right) \end{bmatrix} \begin{bmatrix}\Delta P_L \\ \Delta Q_L\end{bmatrix} \tag{10}$$

where are **I** unit $n\times n$ matrix, $\Delta P_L = [\Delta P_{L1}\ \Delta P_{L2} \cdots \Delta P_{Ln}]^T$, $\Delta Q_L = [\Delta Q_{L1}\ \Delta Q_{L2} \cdots \Delta Q_{Ln}]^T$, $\left.\frac{\partial f_P}{\partial V}\right|_0 = \begin{bmatrix} \frac{\partial f_{P1}}{\partial V_1} & & 0 \\ & \ddots & \\ 0 & & \frac{\partial f_{Pn}}{\partial V_n}\end{bmatrix}_0$.,

$$\left.\frac{\partial f_Q}{\partial V}\right|_0 = \begin{bmatrix} \frac{\partial f_{Q1}}{\partial V_1} & & 0 \\ & \ddots & \\ 0 & & \frac{\partial f_{Qn}}{\partial V_n}\end{bmatrix}_0, \text{ and } T = \begin{bmatrix} T_1 & & 0 \\ & \ddots & \\ 0 & & T_n \end{bmatrix}$$

In mentioned mathematical model Eq. (10), appearance of voltage collapse is assessed basing on state matrix eigenvalues. This, practically means that the answer results from the solution of the following algebraic equation:

$$\det\begin{bmatrix} -T^{-1}\left(I - \left(\frac{\partial f_P}{\partial V}\frac{\partial V}{\partial P_L}\right)_0\right) - \lambda I & T^{-1}\left(\frac{\partial f_P}{\partial V}\frac{\partial v}{\partial Q_L}\right)_0 \\ T^{-1}\left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial P_L}\right)_0 & -T^{-1}\left(I - \left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial Q_L}\right)_0\right) - \lambda I \end{bmatrix} = 0 \tag{11}$$

Using elementary transformations, the system of equations Eq. (11) can be reduced to following, simple form:

$$\det\begin{bmatrix} -T^{-1} - \lambda I & 0 \\ T^{-1}\left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial P_L}\right)_0 & -T^{-1}\left(I - \left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial Q_L}\right)_0 - \left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial P_L}\right)_0 \left(\frac{\partial f_Q}{\partial V}\right)_0^{-1} \left.\frac{\partial f_p}{\partial V}\right|_0\right) - \lambda I \end{bmatrix} = 0 \tag{12}$$

From Eq. (12) is obvious that *n* eigenvalues are always real and negative ($-1/T_1, \ldots, -1/T_n$). For the voltage collapse assessment purpose, only sign of appropriate eigenvalues are needed, i.e. quantification of first *n* eigenvalues is not necessary. Then, problem is reduced to the determination of *n* eigenvalues.

$$A = -T^{-1}\left(I - \left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial Q_L}\right)_0 - \left(\frac{\partial f_Q}{\partial V}\frac{\partial V}{\partial P_L}\right)_0 \left(\frac{\partial f_Q}{\partial V}\right)_0^{-1} \left.\frac{\partial f_P}{\partial V}\right|_0\right) \tag{13}$$

Regarding that $\left.\frac{\partial f_Q}{\partial V}\right|_0$ (i.e. $\left(\frac{\partial f_Q}{\partial V}\right)_0^{-1}$) is diagonal, after some elementary transformations, the matrix **A** can be written in following form:

$$-T^{-1}\left.\frac{\partial V}{\partial Q_L}\right|_0 \left(\left(\left(\frac{\partial V}{\partial Q_L}\right)_0^{-1} - \left.\frac{\partial f_Q}{\partial V}\right|_0\right) - \left(\frac{\partial V}{\partial Q_L}\right)_0^{-1}\left(\frac{\partial V}{\partial P_L}\right)_0 \left.\frac{\partial f_P}{\partial V}\right|_0\right) \tag{14}$$

Starting from the expressions for Q_L and P_L, matrices $\left.\frac{\partial V}{\partial Q_L}\right|_0$ and $\left.\frac{\partial V}{\partial P_L}\right|_0$ can be determined as functions of voltage magnitude V and angle Θ, i.e. $Q_L = g_Q(V,\Theta)$ and $P_L = g_P(V,\Theta)$:

$$\left.\frac{\partial V}{\partial Q_L}\right|_0 = \left(\left.\frac{\partial g_Q}{\partial V}\right|_0 - \left.\frac{\partial g_Q}{\partial \Theta}\right|_0 \left(\frac{\partial g_P}{\partial \Theta}\right)_0^{-1} \left.\frac{\partial g_P}{\partial V}\right|_0\right)^{-1} \tag{15}$$

$$\left.\frac{\partial V}{\partial P_L}\right|_0 = -\left(\left.\frac{\partial g_Q}{\partial V}\right|_0 - \left.\frac{\partial g_Q}{\partial \Theta}\right|_0 \left(\frac{\partial g_P}{\partial \Theta}\right)_0^{-1} \left.\frac{\partial g_P}{\partial V}\right|_0\right)^{-1} \left.\frac{\partial g_Q}{\partial \Theta}\right|_0 \left(\frac{\partial g_P}{\partial \Theta}\right)_0^{-1} \tag{16}$$

In Eq. (14), **T** is diagonal matrix and does not have effect on sign of eigenvalues. For this reason it can be neglected.

Then, respecting Eqs. (15) and (16), following matrix of order *n* can be used for voltage collapse assessment:

$$\mathrm{PQU} = -\left(\left.\frac{\partial g_Q}{\partial V}\right|_0 - \left.\frac{\partial g_Q}{\partial \Theta}\right|_0 \left(\frac{\partial g_P}{\partial \Theta}\right)_0^{-1} \left.\frac{\partial g_P}{\partial V}\right|_0 \right)^{-1} \cdot$$

$$\cdot \left(\left.\frac{\partial g_Q}{\partial V}\right|_0 - \left.\frac{\partial f_Q}{\partial V}\right|_0 + \left.\frac{\partial g_Q}{\partial \Theta}\right|_0 \left(\frac{\partial g_P}{\partial \Theta}\right)_0^{-1} \left(\frac{\partial f_P}{\partial V} - \frac{\partial g_P}{\partial V}\right)_0 \right) \qquad (17)$$

Elements of this matrix are simple to obtain on the basis of known parameters and state variables of the power system. Thus, a relatively simple mathematical model is formed. This model is very suitable for the analysis of the power system voltage (in)stability, because problem is reduced to determination of eigenvalues of a real matrix PQV, of relatively low order (equal to the number of load nodes).

IV. Uniform Movement of Generator Rotors

Voltage (in)stability is practically always performed for characteristic post dynamic quasi-states of the power system. The assumption that the change in power at load nodes coincides with the uniform synchronous generators movements is there completely justified. In practice, this means that synchronous machines participate in total acceleration power P_{ac} according to their inertia constants [9], i.e.

$$\frac{P_{Ti} - P_i}{M_i} = \frac{P_{ac}}{M_s} \ , \qquad (18)$$

where (for *i*-th synchronous machine) are P_{Ti} mechanical power, P_i injected active power, $M_i = (T_{ji}\, S_{ni})/\omega_s$ inertia constant, T_{ji} inertia time constant, S_{ni} nominal power of machine, ω_s - synchronous velocity of rotation, $M_s = \sum_{i=1}^{m} M_i$ and *m* number of synchronous machines.

If participation of M_i in M_s is noted as F_i $(F_i = M_i / M_s ; \sum_{i=1}^{m} F_i = 1)$ and if *m*-th machine is reference one:

$$\frac{P_{Ti} - P_i}{F_i} = \frac{P_{Tm} - P_m}{F_m} \ ; \qquad i = 1,2,\cdots,m-1. \qquad (19)$$

Thus, the condition of uniform movement of generator rotors reduces to the form:

$$R_{Ti} = R_i \ ; \quad i = 1,2,\cdots,m-1 \qquad (20)$$

where:

$$R_{Ti} = F_m P_{Ti} - F_i P_{Tm} \ \text{,and} \ R_i = F_m P_i - F_i P_m \ . \qquad (21)$$

Changes in injected power at *i*-th generator node, as result of load power changes, can be expressed as:

$$\Delta P_i = k_i \Delta\left(P_{loss} + \sum_{j=1}^{n} P_{Lj} \right) ; \ \sum_{i=1}^{m} k_i = 1 \qquad (22)$$

where k_i is incremental coefficient which corresponds to generator production at node *i*, and P_{loss} is total active power loses of system.

Regarding two last expressions, for matrix PQU can be finally written:

$$\mathbf{PQU} = -\mathbf{G}^{-1} \left[\left.\frac{\partial \mathbf{g}_Q}{\partial \mathbf{V}}\right|_0 - \left.\frac{\partial \mathbf{f}_Q}{\partial \mathbf{V}}\right|_0 + \left.\frac{\partial \mathbf{g}_Q}{\partial \Theta}\right|_0 \begin{bmatrix} \frac{\partial \mathbf{g}_P}{\partial \Theta} \\ \frac{\partial \mathbf{g}_R}{\partial \Theta} \end{bmatrix}_0^{-1} \begin{bmatrix} \frac{\partial \mathbf{f}_P}{\partial \mathbf{V}} - \frac{\partial \mathbf{g}_P}{\partial \mathbf{V}} \\ \mathbf{F}\frac{\partial \mathbf{f}_P}{\partial \mathbf{V}} - \frac{\partial \mathbf{g}_R}{\partial \mathbf{V}} \end{bmatrix}_0 \right] \qquad (23)$$

where:

$$\mathbf{G} = \left[\left.\frac{\partial \mathbf{g}_Q}{\partial \mathbf{V}}\right|_0 - \left.\frac{\partial \mathbf{g}_Q}{\partial \Theta}\right|_0 \begin{bmatrix} \frac{\partial \mathbf{g}_P}{\partial \Theta} \\ \frac{\partial \mathbf{g}_R}{\partial \Theta} \end{bmatrix}_0^{-1} \begin{bmatrix} \frac{\partial \mathbf{g}_P}{\partial \mathbf{V}} \\ \frac{\partial \mathbf{g}_R}{\partial \mathbf{V}} \end{bmatrix}_0 \right] \qquad (24)$$

g_R is vector that shows conditions of uniform movement of rotors, $g_R(V, \Theta)$, and **F** is matrix with *n* identical columns:

$$\left[F_m k_1 - F_1 k_m \ \ F_m k_2 - F_2 k_m \ \cdots \ F_m k_{m-1} - F_{m-1} k_m \right]^T \qquad (25)$$

V. Test Example

Presented procedure is applied for voltage collapse assessment of a power system that is shown in Fig. 1 [10]. System consists of 13 nodes, 4 of which are generator nodes. The data in per unit values of the system (generated powers, load powers and nodes voltages) are shown in Table I. The voltages in same table are calculated using Newton-Raphson method.

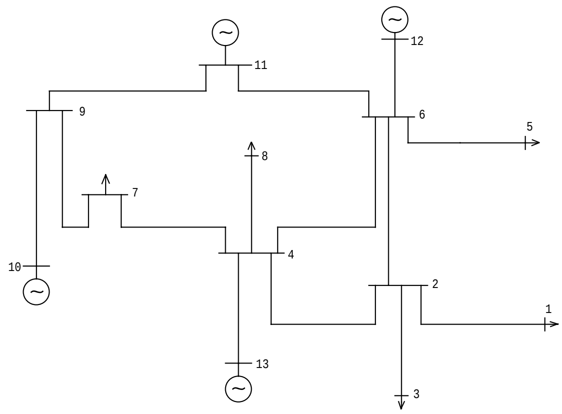

Fig. 1 Test system for voltage collapse assessment

The corresponding incremental factors k_i are: $k_1 = 0.2$, $k_2 = 0.28$, $k_3 = 0.22$, $k_4 = 0.3$, while factors F_i are: $F_1 = 0.15$, $F_2 = 0.3$, $F_3 = 0.25$, $F_4 = 0.3$.

For this initial state of the test network, and for different values of voltage selfregulation coefficients (with assumption that they are the same for all load nodes), voltage collapse assessment is made.

On the basis of results shown at the Tabes I and II following statement can be established: eigenvalues obtained if influence of synchronous generators are not considered are approximately equal to ones obtained when uniform movement of generator rotors is included. Thus, for purpose of voltage collapse fast assessment, it is suitable to use Eq. (17) for calculation matrix **PQU**, because of its simplicity comparing to Eq. (23).

TABLE I
GENERATED POWERS, LOAD POWERS AND VOLTAGES FOR TEST SYSTEM

Node	Generated power	Load power		Voltage magnitude	Voltage phase angle
	P_G(p.u.)	P_L(p.u.)	Q_L(p.u.)	V(p.u.)	($^\circ$)
1		3.5	2.0	0.826466	-24.4156
2		0.0	0.0	0.896634	-16.8234
3		2.12	1.1	0.859977	-23.9969
4		0.0	0.0	0.944272	-10.657
5		1.3	0.8	0.905786	-15.3282
6		0.0	0.0	0.987532	-5.5421
7		2.95	1.4	0.920813	-11.5256
8		1.1	0.7	0.907940	-18.702
9		0.9	0.5	1.027933	5.2696
10	2.7			1.05	10.2098
11	3.0			1.05	6.3277
12	3.2			1.05	0.3605
13	3.452			1.05	0.0

TABLE II
EIGENVALUES OF THE PQU MATRIX WITHOUT CONSIDERATION INFLUENCE OF SYNCHRONOUS GENERATORS

	$k_{pv} = k_{qv} = 2$	$k_{pv} = k_{qv} = 1$	$k_{pv} = k_{qv} = 0$
λ_1	-3.91786	0.004088	1.05967
λ_2	-1.42943	-0.85222	-0.69687
λ_3	-1.20511	-0.92942	-0.85522
λ_4	-1.30729	-0.89426	-0.78309
λ_5	-1.14506	-0.95008	-0.8976
λ_6	-1.0000	-1.0000	-1.0000
λ_7	-1.01299	-0.99553	-0.99083
λ_8	-1.0000	-1.0000	-1.0000
λ_9	-1.0000	-1.0000	-1.0000

TABLE III
EIGENVALUES OF PQU MATRIX WHEN UNIFORM MOVEMENTS OF SYNCHRONOUS GENERATORS IS CONSIDERED

	$k_{pv} = k_{qv} = 2$	$k_{pv} = k_{qv} = 1$	$k_{pv} = k_{qv} = 0$
λ_1	-3.60740	0.0082376	1.15838
λ_2	-1.40995	-0.867497	-0.72057
λ_3	-1.29324	-0.90344	-0.79801
λ_4	-1.17375	-0.93441	-0.85790
λ_5	-1.12872	-0.94556	-0.89901
λ_6	-0.99958	-0.99091	-0.98719
λ_7	-0.99958	-1.00355	-1.00498
λ_8	-1.00008	-1.0000	-1.00004
λ_9	-0.99999	-1.00003	-1.0000

In any of two mentioned cases, when $k_{pv} = k_{qv} = 1$ or $k_{pv} = k_{qv} = 0$, one of eigenvalues is positive what indicate possibility of voltage collapse appearing. Test example verifies the fact that loads with smallest voltage selfregulation coefficients of active and reactive power are critical, i.e. they have larger contribution to voltage collapse appearance.

However, regarding that in this approach generator nodes are modelled only as constant voltage sources, when critical points are determined, we should reconsider using some exact method to test if voltage collapse appears.

VI. CONCLUSION

The simplified linearized dynamic model for fast voltage collapse assessment is formed in this paper. For the case of *n*-load nodes network appropriate transformations prove that instead of *2n*, it is needed to determine only *n* eigenvalues. Additionally, knowing of the values of load time constant is needless also, as it is shown in this paper. Presented linearized dynamical model, because of introduced assumptions, can be used only for fast assessment of voltage collapse. When critical states are identified, other method should be utilised in order to clearly determine whether voltage collapse appears.

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