Calculation of Power Grid Losses Via Node Electric Load Mathematical Model

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Abstract: **A simplified though sufficiently accurate model is presented in the paper of node electric loads in the power grid. Electric load is presented by one constant component and a random process. This random process is modelled by Fourier sequence. The model utilizes representative harmonics only. Modelled loads are used to calculate power losses quickly and fairly accurately.**

Key words: **power losses, electric load**

The main parameter of power grid operation is the level of power loss therein. Loss calculation requires a huge volume of constantly updated data on node electric loads. This data is not always obtainable and, furthermore, its processing involves labour-consuming calculations. With a complex grid configuration, the available fairly accurate methods are inefficient, and approximation methods yield significant error [1]. Calculation process can be eased and accelerated by a comparatively simplified and sufficiently adequate mathematical model of the complex load in the respective node.

Active and reactive node loads have clearly expressed periodicity, with 24- and 168 - hour periods. Within a longer interval they contain a seasonal component as well. Fourier sequence is especially appropriate for there presentation.

Electric power consumption in the node is also characterized by a constant component. It represents the mathematical expectation of active and reactive load, respectively, for a certain time period T and is defined by the expression

$$M(P_i) = \frac{\sum_{t=1}^{T} P_i(t)}{T} \quad , \quad M(Q_i) = \frac{\sum_{t=1}^{T} P_i(t)}{T} \tag{1}$$

where t is the discrete time index.

The complete mathematical model representing electric load in power grid nodes is, as follows:

$$P_i = M(P_i) + \sum_{k=1}^{n} \left[a_{P_{ik}} \sin\omega_k + b_{P_{ik}} \cos\omega_k \right] + V_{P_i}(T)$$

$$Q_i = M(Q_i) + \sum_{k=1}^{n} \left[a_{Q_{ik}} \sin\omega_k + b_{Q_{ik}} \cos\omega_k \right] + V_{Q_i}(T) \tag{2}$$

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where

- i – is the node index;
- k – harmonic index;
- n – number of representative harmonics;
- a_{Pik}, b_{Pik}, a_{Qik}, and b_{Qik} are coefficients of Fourier sequence for active and reactive load, respectively;
- $\omega_k = \frac{2\pi kt}{T}$ is harmonic frequency;
- $V_{Pi}(t)$ and $V_{Qi}(t)$ are random ergodic processes with zero mathematical expectation. These are known as white Gauss noises without consequence [2]. They are justified by the fact that the mathematical model of electric load (2) includes only representative harmonics. They express inaccuracy of load modelling.

The actual electric load in each node is substituted by a representative calculated load based on the mathematical model Eq.(2). These calculated loads are used to define power distribution in the grid and active power and electric power losses.

Active power losses in the grid are obtained as a sum of a certain number of components. The first component is obtained through power distribution, the latter in turn obtained by placing in nodes of mathematical expectations of actual loads only. In order to calculate additional losses from each representative harmonic included in the mathematical model Eq.(2), it is necessary to make two capacity distributions, placing the following loads in nodes

$$S_{1ik} = \frac{a_{P_{ik}}}{\sqrt{2}} + j\frac{a_{Q_{ik}}}{\sqrt{2}}$$

$$S_{1ik} = \frac{b_{P_{ik}}}{\sqrt{2}} + j\frac{b_{Q_{ik}}}{\sqrt{2}} \tag{3}$$

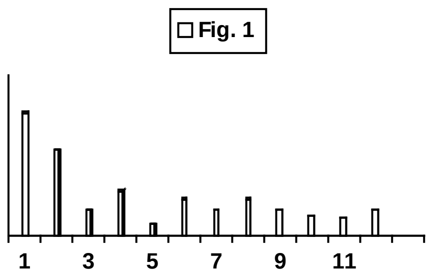
Formula (3) was obtained based on the orthogonal nature of Fourier sequence members. All power loss components are calculated by 2n+1 power distributions, where “n” is the number of representative harmonics. Electric power losses are calculated by the formula

$$\Delta A = \Delta P_m T + \sum_{k=1}^{n} (\Delta P_{ik} + \Delta P_{2k})T + \Delta A_V \tag{4}$$

where: ΔA_V is the calculation error of power losses resulting from non-accounting of all harmonics in node load modelling.

The full model of 24-hour load schedule contains 12 harmonics (fig.1) and that of the weekly load schedule – 84 harmonics (fig.2). Annual load schedule of the complex electric load is decomposed to 4380 harmonics. Inclusion of all harmonics increases many times calculation volume and renders senseless mathematical model application. After obtaining harmonics specter, only the most expressed ones with the greatest amplitude should be selected and included in electric load model. Those are the so-called representative harmonics.

Method efficiency depends mainly on the number of representative harmonics required for the exact modelling of the electric load for the complex consumer. Based on retrospective data on electric loads in main HV grid nodes and application of the procedure for decomposition into Fourier sequence, it was found out that the number of these harmonics is not great. Practically, only two expressed harmonics are enough for fairly accurate modelling of 24-hour load schedule. These are the first and the second harmonics that account 34-
hour and 12-hour periodicity, respectively (fig.1).



Rem. An axe **x** is a frequency specter; an axe **y** is an amplitude

Fig.1. Amplitude – frequency specter of 24-hour electric load in a 110 kV node

For the weekly load schedule of a 110 kV node, the following harmonics are the most influential: 7, 14, 1, 2 (fig.2). The seventh harmonic reflects 24-hour periodicity, the fourteenth one – the 12-hour periodicity; the first one – the weekly periodicity (168 hours), and the second one – internal week (84 hours) periodicity.

When decomposing annual load schedule into Fourier sequence, the most influential harmonics are 1, 4, 56, 112, 365, and 730. The first harmonic reflects annual periodicity, the fourth – the seasonal trend of power consumption, and the rest harmonics reflect weekly, internal weekly, 24-hour, and 12-hour periodicity.

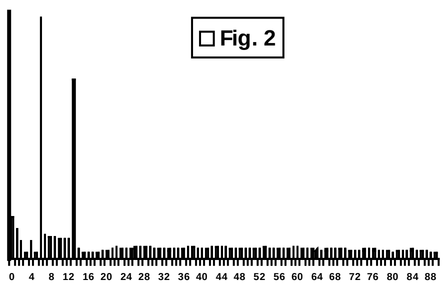


Fig. 2. Amplitude – frequency specter of weekly electric load in a 110 kV node

$S_1 = P_1 + jQ_1$ $\quad$ $S_2 = P_2 + jQ_2$

Fig.3. Model scheme of double side-supply power grid

In order to compare the accuracy of electric load modeling method described herein, electric losses in the model grid were calculated by numerical integration method with quantifying step of 10 minutes. This method was adopted as the accuracy reference standard. Then electric losses were calculated with three variants of electric load modeling. Table1 presents power losses in respective sections of the model grid calculated with the mathematical model Eq.(4) and expressed as percentage of the reference method results for three consecutive days. Column 3 contains power losses calculated with all node loads represented only by their mathematical expectations, in column 4 the losses are calculated by load modeling with mathematical expectations and the first harmonic and in column 5 the losses are obtained via presentation of loads with the first and second harmonics. The results show that the third variant of electric load presentation requiring five power distributions ensures high accuracy of loss calculation

TABLE 1
POWER LOSSES IN RESPECTIVE SECTIONS

Date	Section	ΔA[M(P)] %	ΔA[M(P)+Ix] %	ΔA[M(P)+IIx] %	ΔA_E %
12.1.2000	A-1	83.07	96.07	98.35	100
	1-2	80.71	95.18	97.34	100
	2-B	1.68	97.68	98.49	100
13.1.2000	A-1	83.52	96.03	98.73	100
	1-2	73.07	94.58	94.50	100
	2-B	78.85	96.15	98.02	100
14.1.2000	A-1	82.33	95.54	97.19	100
	1-2	74.67	93.15	97.64	100
	2-B	78.60	94.53	98.14	100

Power losses in the same model grid (fig.3) with actual loads in the same sub-stations for a period of one week, (10 to 16) January 2000, are calculated using four variants of weekly load schedule presentation. The first variant presents loads in p.1 and p.2 by mathematical expectations only. The remaining variants present electric loads with 7, 14, 1, and 2 harmonics added consecutively to the mathematical expectations. The same reference model is used for comparison. The results are shown in Table 2.

TABLE 2
POWER LOSSES

Power losses	%
$\Delta A[M(P)]$	72.14
$\Delta A[M(P) + 7x]$	94.52
$A[M(P) + 7x + 14x]$	97.28
$A[M(P) + 7x + 14x + 1x]$	97.83
$A[M(P) + 7x + 14x + 1x + 2x]$	98.17
ΔA reference	100

Inclusion of 7 and 14 harmonics into the electric load model significantly increases calculation accuracy of power losses, whilst participation of 1 and 2 harmonics does not have such significant impact. In presentation of annual load schedule, it is assumed that the mathematical model (4) should include 5 or 6 harmonics. Due to the relatively consistent nature of electric load, especially with larger number of nodes, the quantity and number of representative harmonics do not change for a long period of time. The proposed method for presentation of electric loads in nodes speeds up the process of power loss calculation in electric grids.

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