Fuzzy Estimation of Criteria Weights in Multiple Criteria Decision Making

Mališa Žižović[1] and Radojica Petrović[2]

Abstract - **A new approach to fuzzy-estimation of criteria weights is explained in this paper taking in account decision-maker's preferences and expert estimations for each criterion. This enables fast obtaining of results for criteria weights and multiple criteria decisions, which improves the efficiency of the team work on a project.**

Keywords - **MCDM, Criteria weight, Fuzzy estimation**

I. INTRODUCTION

Decision making may take some long time if a MCDM method is to be applied consequently by computing precisely all needed parameters within the team work, when both decision maker's preferences and expert judgements are to be observed. The problem is, on one hand, how to give uncertain parameters an exact numerical value, and, on the other hand, how to meet the decision-maker's preferences and the stated criteria in the same time. The aim of this paper is to propose a solution for both with a fuzzy approach improving the solutions given in [1] and [2].

The decision-maker has his subjective estimations based on his preferences, whereas an expert observes a criterion in comparison with the other criteria from the viewpoint of his competence, each criterion having its expert in a project team. The proposed approach makes all of them to participate to defining criteria weight together. The advantage of the fuzzy approach that there is no need to make the estimation precisely enables fast obtaining of results for criteria weights and multiple criteria decisions, which improves the efficiency of the team work on a project.

II. PROCEDURE OF CRITERIA WEIGHT DEFINITION

Step 1. Decision maker defines m criteria $f_1, f_2, \ldots, f_m$ for evaluating the decision alternatives, and eventually states his criteria weights w_k ($k = 1, 2, \ldots, m$) based on his own view of criteria importance for all criteria f_k.

Step 2. Decision maker forms a team of experts with an expert or a group of experts for each criterion.

[1] Mališa Žižović, Technical Faculty, Svetog Save 65, 32000 Čačak, Yugoslavia
[1] Radojica Petrović, Technical Faculty, Svetog Save 65, 32000 Čačak, Yugoslavia, E-mail: radp@ptt.yu

Step 3. All experts or expert groups, having in mind the criterion they are competent for, describe their criteria preferences or their judgement on relative importance of each criterion in comparison with each other, including their conviction about their own judgement.

Step 4. Analyst represents the expert i's fuzzy judgement on relative importance of criterion f_k versus f_j by a fuzzy number F_{kj}^i ($i, j, k = 1, 2, \ldots, m$), which can be either a real number between 0 and 1 or a fuzzy number with a shape as on Fig. 1 or with a trapezoid one or any other shape over the interval $[a, b] \subseteq [0, 1]$, a meaning the lowest preference degree and b the highest. The weakest conviction is represented as 0 and the strongest as 1 on the vertical axis.

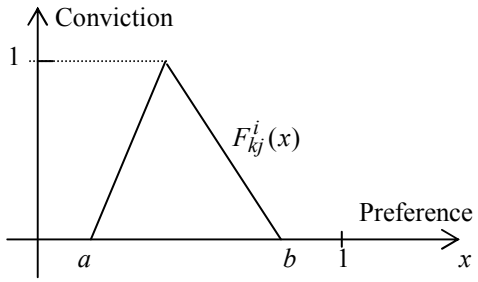

Fig. 1. Interpretation of expert preferences

Step 5. Compute

$$I_{P,kj}^i = \int_a^b F_{kj}^i(x)dx \qquad (1)$$

$$I_{M,kj}^i = \int_a^b xF_{kj}^i(x)dx \qquad (2)$$

for $i, j, k = 1, 2, \ldots, m$.

Then compute

$$p_{kj}^i = \begin{cases} F_{kj}^i & \text{if } F_{kj}^i \text{ is a real number} \\ \dfrac{I_{M,kj}^i}{I_{P,kj}^i} & \text{if } F_{kj}^i \text{ is a fuzzy number} \end{cases} \qquad (3)$$

and compute the sums

$$p_k^i = \sum_{\substack{j=1 \\ j \neq k}}^{m} p_{kj}^i \tag{4}$$

as represented in Table I.

TABLE I

E_i	f_1	f_2	. . .	f_m	p_k^i
f_1	-	p_{12}^i	. . .	p_{1m}^i	p_1^i
f_2	p_{12}^i	-	. . .	p_{2m}^i	p_2^i
. . .	. . .	. . .	. . .	. . .	. . .
f_m	p_{m1}^i	p_{m2}^i	. . .	-	p_m^i

Step 6. Criteria weight coefficient W_k^i that the i-th expert would attribute to the criterion f_k is given by formula (5).

$$W_k^i = \frac{p_k^i}{\sum_{j=1}^{m} p_j^i} \tag{5}$$

Step 7. If the decision maker has not stated his criteria weights for criteria f_k (k = 1, 2, …, m), which means that his own view of criteria importance does not take place in defining criteria weights, then the criteria weight coefficients are defined as an average of the coefficients (5) given by the experts:

$$W_k = \frac{\sum_{i=1}^{m} W_k^i}{m} \tag{6}$$

Otherwise, if the decision maker has stated his criteria weights w_k for all criteria f_k (k = 1, 2, …, m), the criteria weight coefficients are defined by formula (7).

$$W_k^* = \frac{\sum_{i=1}^{m} W_k^i w_k^i}{\sum_{i=1}^{m} w_k^i} \tag{7}$$

III. EXAMPLE

Let us have three criteria f_1, f_2 and f_3, and the corresponding experts or expert groups E_1, E_2 and E_3. Let their criteria preferences are as in TABLES II - IV that can be represented by fuzzy numbers given by Eqs. (8) - (12) for E_1. In a similar way we can define fuzzy numbers for E_2 and E_3. Then we can compute the values p_{kj}^i and p_k^i by Eq. (4), as given in the Tables V - VII.

TABLE II

E_1	f_1	f_2	f_3
f_1	-	high / very high	high
f_2	very low	-	moderate
f_3	low	moderate	-

$$F_{12}^1(x) = \begin{cases} \frac{10}{3}(x-0{,}6), & x \in [0{,}6;\ 0{,}9] \\ -10(x-1), & x \in [0{,}9;\ 1] \\ 0, & x \in [0;\ 0{,}6] \end{cases} \tag{8}$$

$$F_{13}^1(x) = \begin{cases} 5(x-0{,}6), & x \in [0{,}6;\ 0{,}8] \\ -10(x-0{,}9), & x \in [0{,}8;\ 0{,}9] \\ 0, & x \in [0;\ 1] \setminus [0{,}6;\ 0{,}9] \end{cases} \tag{9}$$

$$F_{21}^1(x) = \begin{cases} 5(x-0{,}1), & x \in [0{,}1;\ 0{,}3] \\ -10(x-0{,}4), & x \in [0{,}3;\ 0{,}4] \\ 0, & x \in [0;\ 1] \setminus [0{,}1;\ 0{,}4] \end{cases} \tag{10}$$

$$F_{23}^1(x) = F_{23}^1(x) = \begin{cases} 5(x-0{,}3), & x \in [0{,}3;\ 0{,}5] \\ -5(x-0{,}7), & x \in [0{,}5;\ 0{,}7] \\ 0, & x \in [0;\ 1] \setminus [0{,}3;\ 0{,}7] \end{cases} \tag{11}$$

$$F_{31}^1(x) = \begin{cases} 10(x-0{,}1), & x \in [0{,}1;\ 0{,}2] \\ \frac{-10}{3}(x-0{,}5), & x \in [0{,}2;\ 0{,}5] \\ 0, & x \in [0;\ 1] \setminus [0{,}1;\ 0{,}5] \end{cases} \tag{12}$$

TABLE III

E_2	f_1	f_2	f_3
f_1	-	low / moderate	moderate
f_2	high	-	high
f_3	moderate	low / very low	-

TABLE IV

E_3	f_1	f_2	f_3
f_1	-	high	low / very low
f_2	low / very low	-	low / moderate
f_3	high	high	-

TABLE V

E_1	f_1	f_2	f_3	p_k^1
f_1	-	0,900	0,767	1,667
f_2	0,100	-	0,468	0,568
f_3	0,226	0.620	-	0,846

TABLE VI

E_2	f_1	f_2	f_3	p_k^2
f_1	-	0,133	0,620	1,753
f_2	0,733	-	0,900	1,633
f_3	0,468	0,133	-	0,601

TABLE VII

E_3	f_1	f_2	f_3	p_k^3
f_1	-	0,767	0,100	0,867
f_2	0,100	-	0,200	0,300
f_3	0,700	0,900	-	1,600

Finally, by Eqs. (5) and (6), we obtain

$$W_1 = 0{,}369$$
$$W_2 = 0{,}280$$
$$W_3 = 0{,}351 \qquad (13)$$

IV. CONCLUSION

A method to fast computing criteria weights with fuzzy interpretation of criteria preferences is presented in this paper enabling to meet the decision makers criteria ranking and experts' professional judgement.

REFERENCES

1. M. Žižović, R. Petrović and R. Stanković, "Defining Criteria Weights for Multiple Criteria Estimation of Quality System", *28th Jupiter Conference Proceedings,* pp. 5.17-5.20, Beograd, 2002.
2. M. Žižović, O. Nikolić and R. Petrović, "Project Evaluation with Fuzzy Approach to Criteria Weight Definition", *YUPMA Proceedings*, pp. 62 - 66, Zlatibor, 2002