ON DISTANCE IN POST ALGEBRAS

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Abstract. We define a polynomial d which is a distance in Post algebras. This distance is unique.

Introduction

Let P be an $r-$Post algebra, with underlying chain $C = \{0 = e_0 < e_1 < \ldots < e_{r-1} = 1\}$, where r is an integer and $r \geq 2$. Let $x \vee y$ and xy denote supremum and infimum of the elements x and y.

Theorem 1 [1]. *Every element x of P has a unique representation in disjunctive form*

$$x = \bigvee_{i=0}^{r-1} x^i\, e_i$$

where x^i (Boolean elements, called Postian components of x) satisfy the orthonormality conditions

$$\bigvee_{i=0}^{r-1} x^i = 1 \qquad \text{and} \qquad i \neq j \Rightarrow x^i x^j = 0.$$

If $x \in P$ and there exists an element $\overline{x} \in P$ satisfying the conditions

$$x \vee \overline{x} = 1 \qquad \text{and} \qquad x\,\overline{x} = 0$$

then $\overline{x}$ is called the complement of x.

Theorem 2 [1]. *If f is a Post polynomial in n variables , then*

$$f(x_1, \ldots, x_n) = \bigvee_{(a_1,\ldots,a_n)\in C^n} f(a_1, \ldots, a_n) x_1^{a_1} \cdots x_n^{a_n}.$$

Subject Classification: 03 G 20.
Key words and phrases. Post algebra; distance.

Results

Theorem 3. *Let x and y be elements of Post algebra P. Then*

$$x = y \Leftrightarrow \bigvee_{i=0}^{r-1} (\overline{x^i} y^i \vee x^i \overline{y^i}) = 0.$$

Definition 1. If x and y are elements of Post algebra P then

$$x + y = \bigvee_{i=0}^{r-1} (\overline{x^i} y^i \vee x^i \overline{y^i}).$$

Theorem 4. *The function $d : P^2 \to P$ defined by $d(x, y) = x + y$*
satisfies the conditions
(i) $\quad d(x, y) = 0 \Leftrightarrow x = y$
(ii) $\quad d(x, y) = d(y, x)$
(iii) $\quad d(x, z) \leq d(x, y) \vee d(y, z)$
for all $x, y, z \in P$, i.e. d is a distance.

Theorem 5. *In a Post algebra the unique distance expressed by polynomial is*

$$d(x, y) = x + y.$$

References

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